23/01/2025
Evening

Memory Based
Answers & Solutions

Time : 3 hrs. M.M. : 300

for

JEE (Main)-2025 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains **Three Parts**. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is **Mathematics**. Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt all questions.

(6) **Section - A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section - B (21 – 25) contains** 5 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

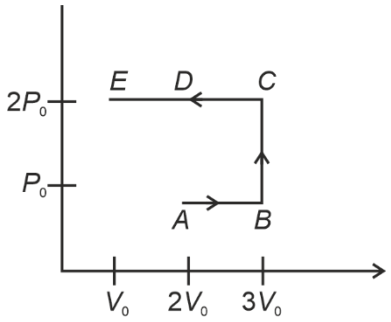
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Find total work done from *A* to *E*.



(1) $-3\ P_0V_0$ (2) $3\ P_0V_0$

(3) $3\ P_0V_0$ (4) $5\ P_0V_0$

Answer (1)

Sol. Work done = Area under curve

$= -4\ P_0V_0 + P_0V_0$

$= -3\ P_0V_0$

2. **Statement 1:** Graph of frequency *f* of X ray and atomic number Z of heavy nucleus is straight line, in X ray emission.

Statement 2: Graph of square root of frequency $\sqrt{f}$ of X ray and atomic number Z of heavy nucleus is straight line, in X ray emission.

(1) Statement 1 is correct and statement 2 is correct

(2) Statement 1 is incorrect and statement 2 is correct

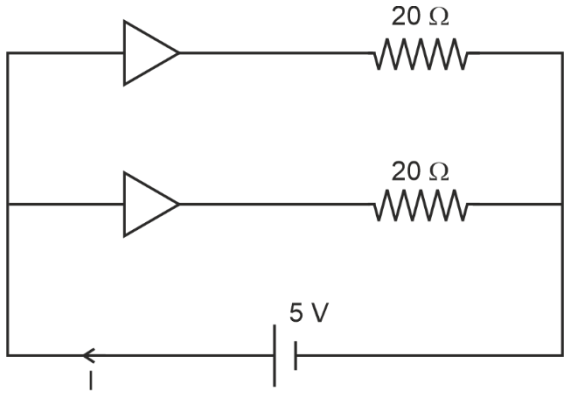
(3) Statement 1 is correct and statement 2 is incorrect

(4) Statement 1 is incorrect and statement 2 is incorrect

Answer (2)

Sol. Mosley law $\sqrt{f} = a(z-b)$

3. Two ideal diodes are connected in circuit as shown. Find current through battery.



(1) 0.3 A

(2) 1 A

(3) 0.5 A

(4) 0.25 A

Answer (3)

Sol. Both diode are in forward bias.

$$I = \frac{5}{20} + \frac{5}{20}$$

$I = 0.5$ A

4. In a series LCR circuit, inductance $L = 100$ mH and capacitance $C = 1$nF. The angular frequency of the source when current has maximum amplitude in the circuit is

(1) $\frac{10^4}{2\pi}$ rad/s (2) $\frac{10^5}{2\pi}$ rad/s

(3) 10^5 rad/s (4) 10^6 rad/s

Answer (3)

Sol. The current is maximum at resonance,

$$\Rightarrow w = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(100\times10^{-3}\text{H}\right)\left(1\times10^{-9}\text{F}\right)}}$$

$= 10^5$ rad/s

5. Two charges +7 C and –4 C are located at (–7, 0, 0) and (7, 0, 0), find electrostatic potential energy of the system. $\left(k = \frac{1}{4\pi \in_0} = 9\times10^9 \text{ SI units}\right)$

(1) -6×10^9 J (2) -18×10^9 J

(3) 18×10^9 J (4) 6×10^9 J

Answer (2)

Sol. $E = \frac{Kq_1q_2}{r}$

$= \frac{-9\times10^9 \times 7 \times 4}{14}$

$= -18 \times 10^9$ J

6. A satellite is nine times closer to earth compared to moon. Time period of moon is 27 days then period of satellite is

(1) 3 days

(2) 9 days

(3) 1 day

(4) $3\sqrt{3}$ days

Answer (3)

Sol. $\frac{T_m^2}{T_s^2} = \frac{r_m^2}{\left(\frac{r_m}{9}\right)^3} = 9^3$

$\Rightarrow T_s = \frac{T_m}{9^{3/2}} = 1$ day

7. A concave mirror of focal length f dipped into a fluid of refractive index μ. The new focal length mirror is

(1) μf (2) f

(3) $\frac{f}{\mu - 1}$ (4) $\frac{f}{\mu}$

Answer (2)

Sol. Focal length of spherical mirror does not depends on medium in which it placed.

8. In an electromagnetic wave of frequency 20 MHz, value of electric field is 9.3 V/m, then magnitude of magnetic field at that instant is

(1) 3.1×10^8

(2) 27.9×10^8

(3) 3.1×10^{-8}

(4) 18.6×10^{-6}

Answer (3)

Sol. $E = CB$

$\Rightarrow B = \frac{9.3}{3\times10^8} = 3.1\times10^{-8}\ T$

9. Match the correct dimensions

(a)	Magnetic field	(i)	ML^2
(b)	Permittivity of free space	(ii)	$M^{-1}L^{-3}T^4A^2$
(c)	Moment of inertia	(iii)	$MT^{-2}A^{-1}$
(d)	Velocity	(iv)	LT^{-1}

(1) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (i), (d) $\rightarrow$ (iii)

(2) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (iv)

(3) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (i), (d) $\rightarrow$ (iv)

(4) (a) $\rightarrow$ (i), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (iii), (d) $\rightarrow$ (iv)

Answer (3)

Sol. $qvB = F$

$B = \frac{MLT^{-2}}{A}$

$MT^{-2}A^{-1}$

$\frac{q}{\varepsilon_0} = EA$

$\frac{1}{\varepsilon_0} = \frac{MLT^{-2}}{AT}\frac{L^2}{AT}$

$\varepsilon_0 = \frac{A^2T^2}{ML^3T^{-2}}$

10. The temperature of a body of mass m and specific heat capacity s is raised slowly from T_1 to T_2. The change is entropy of the system is

(1) $ms\ln\left(\frac{T_2}{T_1}\right)$

(2) ms

(3) $ms\ln\left(\frac{T1}{T_2}\right)$

(4) Zero

Answer (1)

Sol. $d(\text{Entrophy})_{sys} = \frac{dQ}{T}$

$$(\Delta\text{Entropy})_{sys} = \int_{T_1}^{T_2}\frac{dQ}{T}$$

$$= ms\int_{T_1}^{T_2}\frac{dT}{T}$$

$$= ms\ \ln\frac{T_2}{T_1}$$

11. A moving coil galvanometer with coil resistance $G = 30\ \Omega$, shows full scale deflection when the current through it is 20 mA. The galvanometer is converted to an ammeter of range 3 A by using a shunt resistance S, then resistance S is

(1) 0.2 Ω

(2) 2 Ω

(3) 0.8 Ω

(4) 1.2 Ω

Answer (1)

Sol. $i_g G = IS$

$$S = \frac{i_g G}{I}$$

$$= \frac{(20\times10^{-3}\,\text{A})(30\ \Omega)}{(3\,\text{A})}$$

$$= \frac{1}{5}\Omega$$

12. Torque on a uniform disk of mass 2 kg, radius 1 m is given as $\tau(t) = 5t^2 - 8t$. If the disk was initially at rest, find power by torque at t = 1s.

(1) 5 W

(2) 3 W

(3) 7 W

(4) 9 W

Answer (3)

Sol. $\rho = \vec{\tau}\cdot\vec{\omega}$

$\tau = I\alpha$

$I = \frac{MR^2}{2} = 1$

$\alpha = 5t^2 - 8t$

$\omega = \frac{5}{3}t^3 - 4t^2$

At t = 1, $\omega = \frac{5}{3} - 4 = -\frac{7}{3}$

$t(3) = 5 - 8 = -3$

$\rho = \vec{\tau}\cdot\vec{\omega}$ = 7 W

13. During charging of capacitor of 2.5 μF in DC circuit, the displacement current is found to be 0.25 mA then find rate of change of voltage V w.r.t. time $\frac{dV}{dt}$.

(1) 1 V/s

(2) 10 V/s

(3) 100 V/s

(4) 0.1 V/s

Answer (3)

Sol. $V = \frac{q}{C}$

$$\frac{dV}{dt} = \frac{i}{C} = \frac{0.25\times10^{-3}}{2.5\times10^{-6}}$$

$$\frac{1000}{10}$$

14. In a photoelectric experiment, the stopping potential of 2 V, the work function of metal is 2.14 eV. Find the wavelength of incident light (given hc = 1242 eV-nm)

(1) 200 nm

(2) 300 nm

(3) 600 nm

(4) 400 nm

Answer (2)

Sol. $E = 2 + 2.14 = 4.14$ eV

$$E = \frac{hc}{\lambda}$$

$$\therefore \quad \lambda = \frac{1242}{4.14} = 300 \text{ nm}$$

15. The equation of wave travelling in a medium is given by $y(x, t) = 4.0\sin(20 \times 10^{-3}x + 600t)$mm. The velocity of wave is $n \times 10^4$ m/s. Find n.

(1) 7

(2) 5

(3) 4

(4) 3

Answer (4)

Sol. Velocity of wave $= \frac{\text{Coefficient of } t}{\text{Coefficient of } x}$

$$= \frac{600}{20\times10^{-3}}$$

$$= 3\times10^4 \text{ m/s}$$

$n = 3$

16.

17.

18.

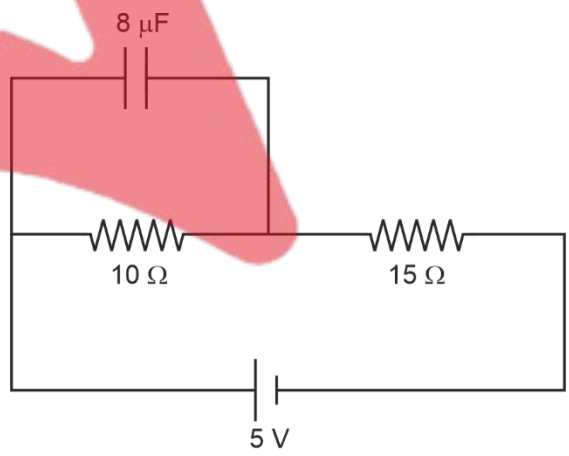
19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Find charge on the capacitor (in μF) at steady state.



Answer (16)

Sol. $i = \frac{5}{25} = \frac{1}{5}$ A

$V = 10 \times \frac{1}{5} = 2$ V

$Q = CV = 8 \times 2\ \mu F$

$= 16\ \mu F$

22. A particle of mass m is projected at angle 60° with horizontal. If initial kinetic energy is KE_0 and kinetic energy at maximum height is $\frac{KE_0}{x}$, find value of x.

Answer (4)

Sol. $KE_0 = \frac{1}{2} mV^2$

$KE = \frac{1}{2} m(V\cos 60°)^2$

$= \frac{1}{2} m \frac{V^2}{4}$

$x = 4$

23. The energy in a system varies with position and time as $E(x, t) = x^3 e^{-\beta t}$, where $\beta = 0.3\ sec^{-1}$. Given that the percentage error in x = 1.2% and that the % error in t = 1.6%, find the maximum % error in E at t = 5 sec.

Answer (6)

Sol. $E = x^3 e^{-\beta t}$

$\ln E = 3\ln x - \beta t$

$\frac{1}{E} dE = \frac{3}{x} dx - \beta\, dt$

$\left(\frac{dE}{E} \times 100\right) = 3\left(\frac{dx}{x} \times 100\right) + 100\beta\, dt$

% error in E = $3(1.2\%) + (100)(0.3)\left(\frac{1.6}{100} \times 5\right)\%$

% error in E = 6%

24. For a prism, the minimum deviation is equal to the angle of prism. If the refractive index is $\sqrt{3}$, find the minimum deviation (in degrees)

Answer (60)

Sol. $\mu = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\mu = 2\cos\left(\frac{A}{2}\right)$

$\frac{A}{2} = 30°$

$A = 60°$

$\therefore\ \delta = 60°$

25. A satellite of mass m is moving in circular orbit at a heigh R from surface of Earth (mass M, radius R). If the angular momentum of the satellite is $m\sqrt{NGMR}$, find N.

Answer (2)

Sol. $L = mvr$

$= m\sqrt{\frac{GM}{2R}}\ 2R$

$= m\sqrt{2GMR}$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The correct order of melting points of Group-14 elements is

   (1) C > Si > Ge > Sn > Pb

   (2) Si > C > Ge > Sn > Pb

   (3) Ge > Sn > C > Si > Pb

   (4) C > Si > Ge > Pb > Sn

Answer (4)

Sol. Melting Points (in K)

C	Si	Ge	Sn	Pb
4373	1693	1218	505	600

Correct order will be

C > Si > Ge > Pb > Sn

2. α-helix protein and β-pleated sheets protein belong to which one of the following types of protein?

   (1) Primary

   (2) Secondary

   (3) Tertiary

   (4) Quarternary

Answer (2)

Sol. α-helix and β-pleated sheet structure belong to secondary protein or 2° protein.

3. What will be effect on pH of water when it is heated?

   (1) Increase

   (2) Decrease

   (3) Remains same

   (4) pH first increases then decreases

Answer (2)

Sol. As water is heated degree of ionisation increases $[H^+]$ ion concentration increases pH decreases.

4. Match the following List I with List II.

List-I (Alloys)		List-II (Metals)	
A.	Bronze	(i)	Fe, Cr, and Ni
B	Stainless steel	(ii)	Cu and Sn
C	UK Gold Coin	(iii)	Cu and Zn
D	Brass	(iv)	Au, Ag, Cu, Zn and Ni

   (1) A-(ii), B-(i), C-(iv), D-(iii)

   (2) A-(iii), B-(iv), C-(i), D-(ii)

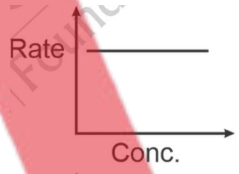
   (3) A-(iv), B-(iii), C-(ii), D-(i)

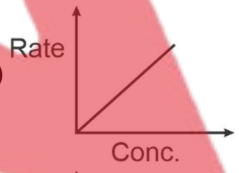
   (4) A-(i), B-(ii), C-(iii), D-(iv)

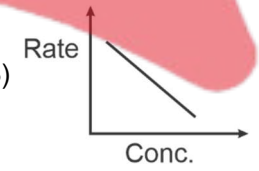
Answer (1)

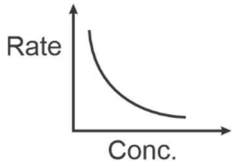
Sol.
- Bronze is Made up of copper and Tin
- Stainless steel is made up of Fe, Cr and Ni
- UK Gold coin is made up of – Au (91%), Ag, Cu, Zn and Ni.
- Brass is made up of Copper and Zinc.

5. Which one of the following plots represents zero order reaction?

(1) 

(2) 

(3) 

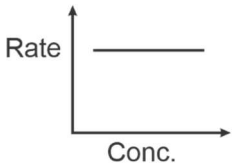
(4) 

Answer (1)

Sol. The rate of a zero order reaction remains constant throughout the reaction.

Rate = k [Reactant]°

Plot of rate vs conc. of reactant is linear parallel to x-axis.



6. By using relation

$\Delta G = \Delta H - T\Delta S$

Which of the following relation is incorrect for spontaneous reaction at a given temperature?

(1) $\Delta H > 0, \Delta S > 0$ (2) $\Delta H > 0, \Delta S < 0$

(3) $\Delta H < 0, \Delta S > 0$ (4) $\Delta H < 0, \Delta S < 0$

Answer (2)

Sol. If $\Delta H > 0, \Delta S > 0$, then process is spontaneous at high temperature

If $\Delta H > 0, \Delta S < 0$, process is always non spontaneous

If $\Delta H < 0$ and $\Delta S > 0$, then process is always spontaneous

If $\Delta H < 0$ and $\Delta S < 0$, then process is spontaneous at low temperature.

7. **Statement-I :** For a particular shell, maximum number of orbitals is n^2.

Statement-II : For a subshell, possible orientations lies between –l to +l including zero.

(1) Statement-I and Statement -II both are correct

(2) Statement-I and Statement-II both are incorrect

(3) Statement-I is correct, Statement-II is incorrect

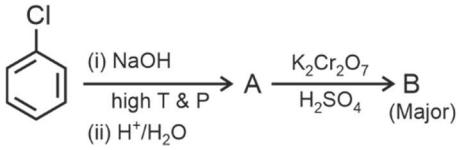
(4) Statement-I is incorrect, Statement-II is correct

Answer (1)

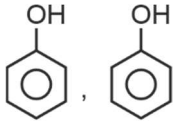
Sol. If shell number is 2, then number of orbitals = $2^2 = 4$

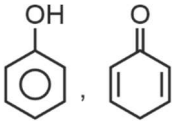
$2s \rightarrow \square$; $2p \rightarrow \square\square\square = 4$

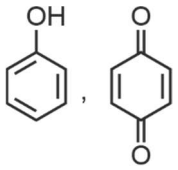
If l = 2, then value lies between –2, –1, 0, +1, +2.

8. 

Predict A and B?

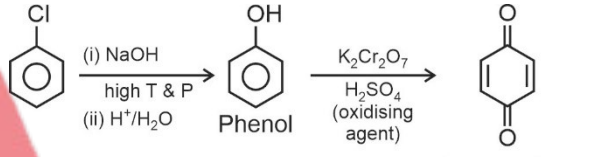
(1) 

(2) 

(3) 

(4) None of these

Answer (3)

Sol. 

p-Benzoquinone

9. Which of the following complex has d^4 configuration?

(1) $[Fe(CN)_6]^{3-}$ (2) $[MnF_6]^{3-}$

(3) $[Co(CN)_6]^{3-}$ (4) $[CoCl_4]^{2-}$

Answer (2)

Sol. $[Fe(CN)_6]^{3-} \Rightarrow Fe^{3+} \Rightarrow 3d^5$ configuration

$[MnF_6]^{3-} \Rightarrow Mn^{3+} \Rightarrow 3d^4$ configuration

$[Co(CN)_6]^{3-} \Rightarrow Co^{3+} \Rightarrow 3d^6$ configuration

$[CoCl_4]^{2-} \Rightarrow Co^{2+} \Rightarrow 3d^7$ configuration

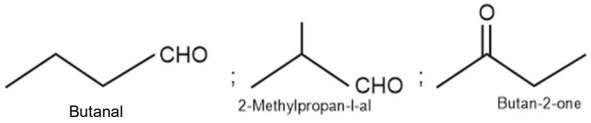
10. The total number of isomers possible (aldehyde & ketones) for C_4H_8O are

(1) 3 (2) 4

(3) 5 (4) 6

Answer (1)

Sol.



11. Consider the following reaction

$X_2Y(s) \rightleftharpoons X_2(g)+\frac{1}{2}Y_2(g)$. If α is the degree of dissociation. Calculate K_P in terms of P (total pressure)

(1) $K_P=\frac{2P^{3/2}}{3^{3/2}}$ (2) $K_P=\frac{2P^{3/2}}{3}$

(3) $K_P=\sqrt{\frac{2P}{3}}$ (4) $K_P=\frac{\sqrt{2}P}{3}$

Answer (1)

Sol. $X_2Y(s) \rightleftharpoons X_2(g)+\frac{1}{2}Y_2(g)$

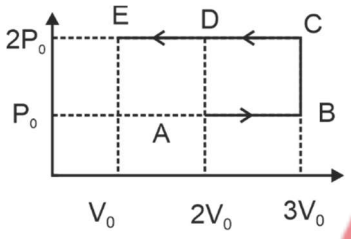
$t_{eq.}$ P' $\frac{P'}{2}$

$P_T=\frac{3P'}{2} \Rightarrow P'=\left(\frac{2P_T}{3}\right)$

$K_P=\left(P_{X_2}\right)\left(P_{Y_2}\right)^{1/2}$

$K_P=P'\left(\frac{P'}{2}\right)^{\frac{1}{2}}=\frac{(P')^{3/2}}{\sqrt{2}}=\frac{\left(\frac{2P}{3}\right)^{3/2}}{\sqrt{2}}=\frac{2P^{3/2}}{3^{3/2}}$

12. An ideal gas undergoes following process from $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E$. Find work done.



(1) $-3P_0V_0$

(2) $2P_0V_0$

(3) $3P_0V_0$

(4) $\frac{3P_0V_0}{2}$

Answer (3)

Sol. |W| = Area under the curve

$= 2P_0 \times [3V_0 - V_0] - P_0 \times [3V_0 - 2V_0]$

$= 3P_0V_0$

Net work is done on the system.

13. When a non-volatile solute (A) is added to a volatile solvent, the vapour pressure of solvent decreases by 10 mm Hg. Mole fraction of solute in solution is 0.2. If another non-volatile solute (B) is further added to the same solution and vapour pressure of solution decreases by 20 mm Hg, calculate mole fraction of 2nd solute in the final solution.

(1) 0.3

(2) 0.4

(3) 0.5

(4) 0.6

Answer (3)

Sol. For solute (A),

$X_A = 0.2$

$\Delta P = 10$ mm Hg

Let $P^o_{solvent}$ be vapour pressure of pure solvent

$\frac{\Delta P}{P^o_{solvent}} = X_A$

$P^o_{solvent} = \frac{10}{0.2} = 50$ mm Hg

For solute (B),

Let number of moles of solvent be 0.8 and n_B be the number of moles of (B) added, then

$X_B = \frac{n_B}{0.2+0.8+n_B} = \frac{n_B}{1+n_B}$

$\Delta P' = 10 + 20 = 30$ mm Hg

$$\frac{\Delta P'}{P^{o}_{solvent}} = \frac{0.2 + n_B}{1 + n_B}$$

$$\frac{30}{50} = \frac{0.2 + n_B}{1 + n_B}$$

$\Rightarrow$ $n_B = 1$

$$X_B = \frac{1}{1+1} = 0.5$$

14. Consider the following E° values of given half cell

$E^{\circ}_{Ag^+/Ag} = 0.8\ V, E^{\circ}_{Zn^{2+}/Zn} = -0.76\ V$

$E^{\circ}_{Cu^{2+}/Cu} = 0.34\ V, E^{\circ}_{Mg^{2+}/Mg} = -2.36\ V$

Then which of the following will have the most negative value of $\Delta G°$?

(1) $Zn|Zn^{2+}||Cu^{2+}|Cu$

(2) $Mg|Mg^{2+}||Ag^{+}|Ag$

(3) $Mg|Mg^{2+}||Zn^{2+}|Zn$

(4) $Cu|Cu^{2+}||Ag^{2+}|Ag$

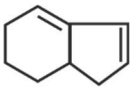
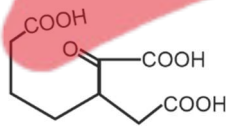
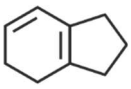
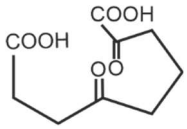
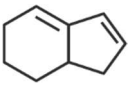
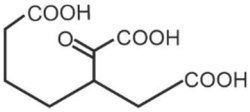
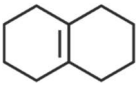
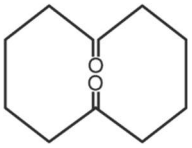
Answer (2)

Sol. The cell which has most positive value of E°_{Cell} will have most negative value of $\Delta G°$.

$E^{o}_{cell} = E^{o}_{Ag^+|Ag} - E^{o}_{Mg^{2+}|Mg} = 0.8 - (-2.36) = 3.16V$

$\Delta G° = -nF\,E^{\circ}_{Cell}$

15. Match the following

	Reactant		Product
(A)		(i)	
(B)		(ii)	
(C)		(iii)	
(D)		(iv)	

Give correct product of oxidative ozonolysis (O_3/H_2O)

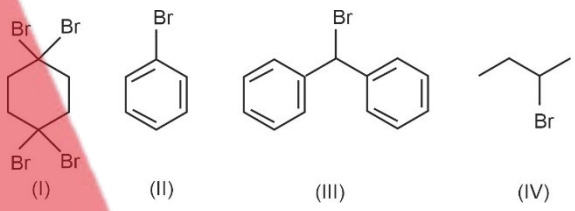
(1) A-ii, B-i, C-iii, D-iv (2) A-i, B-ii, C-iii, D-iv

(3) A-i, B-ii, C-iv, D-iii (4) A-i, B-iv, C-ii, D-iii

Answer (2)

Sol.

16. Arrange the following compounds in the decreasing order of rate of hydrolysis



(1) (I) > (II) > (III) > (IV)

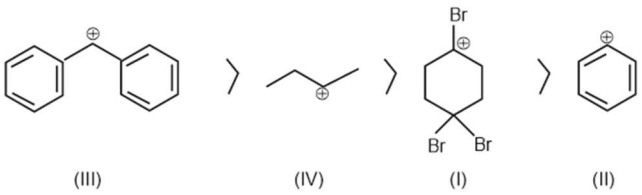
(2) (III) > (IV) > (I) > (II)

(3) (IV) > (III) > (I) > (II)

(4) (III) > (II) > (IV) > (I)

Answer (2)

Sol. Rate of hydrolysis of the given compounds will be decided on the basis of stability of carbocation intermediate. Higher the stability of carbocation, higher will be the rate of hydrolysis of the parent compound. Stability order of carbocations obtained from the given compounds.



17. 0.01 mole of an organic compound (Hydrocarbon) gives 1.76 gm CO_2 and 0.9 gm H_2O on complete combustion. Find out chemical formula of compound.

(1) C_3H_8

(2) C_4H_{10}

(3) C_5H_{12}

(4) C_6H_{14}

Answer (2)

Sol. $C_XH_Y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

$x = \frac{0.04}{0.01} = 4$

$\frac{y}{2} = 5$

y = 10

Formula is C_4H_{10}

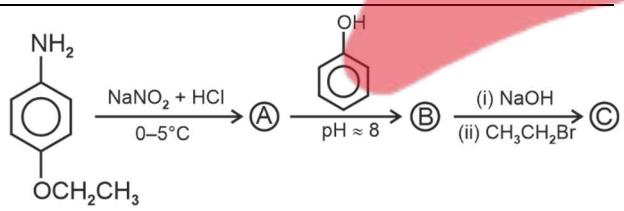
18.

19.

20.

SECTION - B

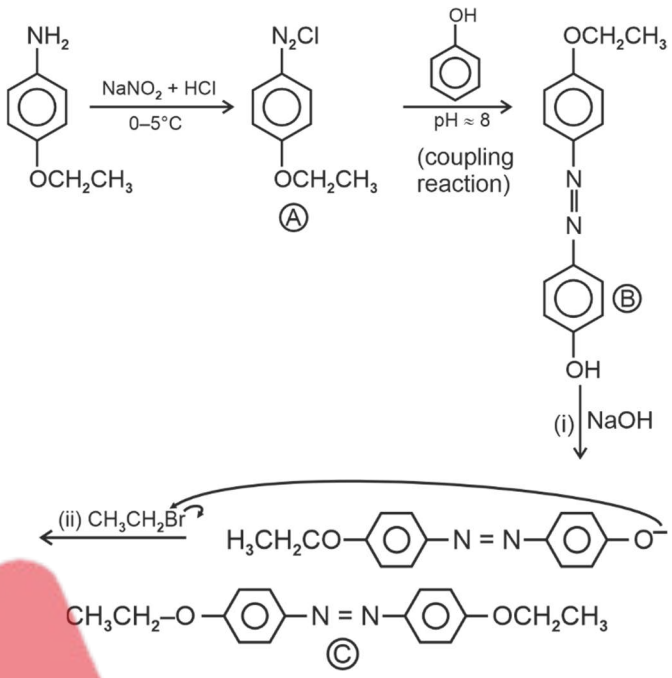
Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. 

Number of sp^3 hybridised carbon atoms in C is :

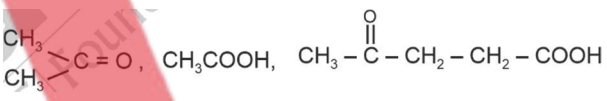
Answer (4)

Sol.



Total 4 sp^3 hybridised carbon atoms are present in Ⓒ

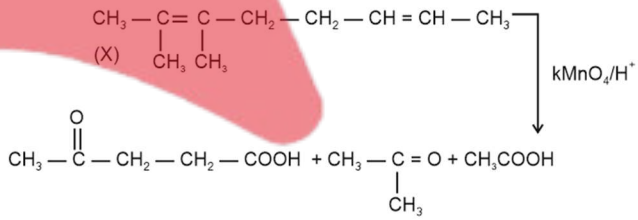
22. A compound X consumes two moles of H_2 and when 'X' heated with $KMnO_4/H^+$ gives



Number of σ bonds in X are _____.

Answer (27)

Sol. Compound X has 27 σ-bonds



23.

24.

25.

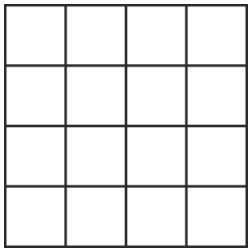
MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If a square is divided in 4 × 4 squares.



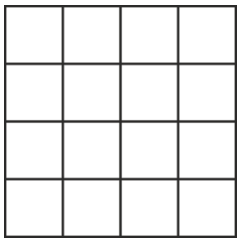
If two squares are chosen randomly then the probability that the squares doesn't share common side is

(1) $\frac{3}{5}$ (2) $\frac{4}{5}$

(3) $\frac{3}{20}$ (4) $\frac{7}{10}$

Answer (2)

Sol.



Total = $^{16}C_2$

Required ways = Total – (adjacent squares)

= $^{16}C_2$ – [3 pairs in vertical and horizontal for each row and column]

= $^{16}C_2$ – [3 × 4 + 3 × 4]

Probability = $\frac{^{16}C_2 - 24}{^{16}C_2} = \frac{120-24}{120} = \frac{96}{120} = \frac{4}{5}$

2. If $I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$, then the value of definite integration $\int_0^{2I} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ is

(1) $\frac{\pi}{16}$ (2) $\frac{\pi^2}{16}$

(3) $\frac{\pi}{8}$ (4) $\frac{\pi^2}{8}$

Answer (2)

Sol. $I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$

$\int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} (1) dx = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

$\Rightarrow 2I = \frac{\pi}{2}$

$\Rightarrow I_1 = \int_0^{2I} \frac{x \sin x \cos x}{(\sin^4 + \cos^4 x)} dx = \int_0^{2I} \frac{x \sin x \cos x}{\sin^4 + \cos^4 x} dx$

$\Rightarrow I_1 = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - 2\right) \cos \sin x dx}{\cos^4 x + \sin^4 x}$

Adding,

$2I_1 = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2} \cos x \sin x dx}{(\sin^4 x + \cos^4 x)}$

$(\sin^4x + \cos^4x) = (\sin^2x + \cos^2x)^2 - 2\sin^2x\cos^2x$

$$= \left(1 - \frac{\sin^2 2x}{2}\right) = \frac{2 - \sin^2 x}{2}$$

$$= \frac{1 - \cos^2 2x}{2}$$

$$2I = \frac{\pi}{4}\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\frac{1+\cos^2 2x}{2}} dx$$

$$\Rightarrow \quad 2I = \frac{\pi}{4}\int_{-1}^{1} \frac{dt}{(1+t^2)}, \quad t = \cos 2x$$

$$\Rightarrow \quad I = \frac{\pi}{8} \cdot \left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right) = \frac{\pi^2}{16}$$

3. Consider the terms 8, 21, 34, 47, …, 320. The variance of the given data set is

(1) 8788 (2) 8614

(3) 720 (4) 9402

Answer (1)

Sol. μ = Mean = $\frac{8 + 21 + \ldots 320}{25} = \frac{25}{2}\frac{(8 + 320)}{25} = 164$

Variance = $\frac{8^2 + 21^2 + \ldots 320^2}{25} - (164)^2 = 8788$

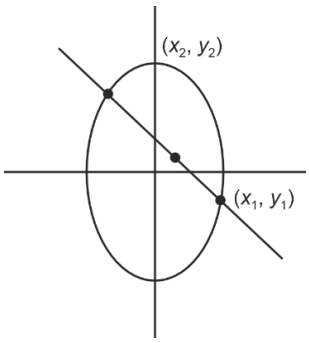
4. Let $M\left(\frac{1}{2}, 1\right)$ be the mid-point of a chord to the Ellipse $\frac{x^2}{2} + \frac{y^2}{4} = 1$, then the length of chord is

(1) $\frac{2}{3}\sqrt{5}$ (2) $\frac{\sqrt{5}}{3}$

(3) $2\sqrt{\frac{5}{3}}$ (4) $\frac{\sqrt{5}}{2}$

Answer (3)

Sol. $\frac{x^2}{2} + \frac{y^2}{4} = 1$



The equation of chord bisected at $\left(\frac{1}{2}, 1\right)$ will be $T = S_1$.

$$\Rightarrow \frac{x}{2}\left(\frac{1}{2}\right) + \frac{y}{4}(1) - 1 = \frac{\left(\frac{1}{2}\right)^2}{2} + \frac{1^2}{4} - 1$$

$$\Rightarrow \frac{x}{4} + \frac{y}{4} = \frac{1}{8} + \frac{1}{4}$$

$$\Rightarrow x + y = \frac{3}{2} \Rightarrow x_1 + y_1 = \frac{3}{2}$$

$$x_2 + y_2 = \frac{3}{2}$$

$$\Rightarrow (x_1 - x_2) + (y_1 - y_2) = 0$$

$$\Rightarrow (x_1 - x_2)^2 = (y_1 - y_2)^2$$

$2x^2 + y^2 = 4$

$$2x^2 + \left(\frac{3}{2} - x\right)^2 = 4$$

$$3x^2 - 3x + \frac{9}{4} - 4 = 0$$

$$\Rightarrow 3x^2 - 3x - \frac{7}{4} = 0$$

$$x_1 + x_2 = 1$$

$$x_1.x_2 = \frac{-7}{12}$$

$$(x_1 - x_2)^2 = (x_1 + x_2) - 4x_1x_2$$

$$= 1 + \frac{28}{72} = \frac{10}{3}$$

$$\Rightarrow \text{Length of chord} = \sqrt{\frac{10}{3} + \frac{10}{3}} = \sqrt{\frac{20}{3}} = 2\sqrt{\frac{5}{3}}$$

5. If the square of the shortest distance between the lines $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z+3}{-3}$ and $\frac{x+1}{2}=\frac{y+3}{4}=\frac{z+5}{-5}$ is $\frac{m}{n}$ (where m, n are coprime number) then $m+n$ equals to

(1) 6 (2) 9

(3) 21 (4) 14

Answer (2)

Sol. $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z+3}{-3}$

$\frac{x+1}{2}=\frac{y+3}{4}=\frac{z+5}{-5}$

$a_1 = (2, 1, -3)$

$a_2 = (-1, -3, -5)$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$

$2\hat{i} - \hat{j}$

$(SD)^2 = \left|\frac{(a_2 - a_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|^2$

$= \left|\frac{(3\hat{i}+4\hat{j}+2k)\cdot(2\hat{i}-\hat{j})}{\sqrt{4+1}}\right|^2$

$= \left(\frac{2}{\sqrt{5}}\right)^2 = \frac{4}{5}$

$m + n = 9$

6. A rod of length 8 units having two end points always lie on $x - y + 2 = 0$ and $x + y + 2 = 0$. A point P divide this line in ratio 2 : 1. Then locus of P is

(1) $9x^2 + 9y^2 + 36x - 28 = 0$

(2) $8x^2 + 8y^2 + 36x + 27 = 0$

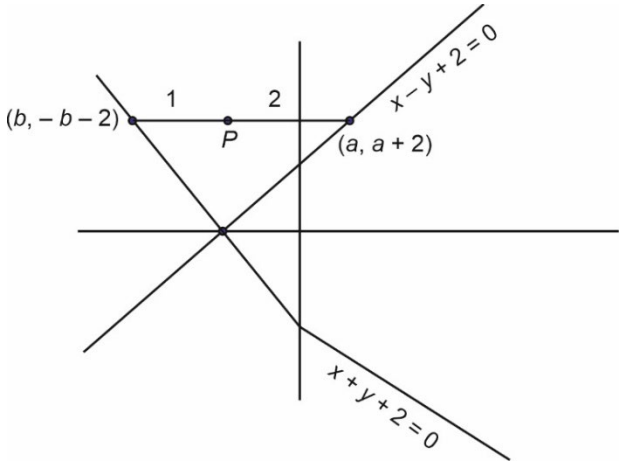
(3) $9x^2 + 9y^2 - 36x + 28 = 0$

(4) $8x^2 + 8y^2 - 36x - 27 = 0$

Answer (1)

Sol. $x - y + z = 0$

$x + y + z = 0$



$PQ = 8$

$\Rightarrow (a-b)^2 + (a+2+b+2)^2 = 64$

$(a-b)^2 + (a+b+4) = 64$

$2a^2 + 2b^2 - 2ab + 16 + 2(ab + 4b + 4a) = 64$

$\Rightarrow 2a^2 + 2b^2 + 8a + 8b = 48$

$\Rightarrow a^2 + b^2 + 4a + 4b - 24 = 0 = (a+2)^2 + (b+2)^2 = 32$

$h = \frac{2b+1(a)}{3}, K = \frac{2(-2-b)+1(a+2)}{3}$

$3h = 2b + a$, $3K = a - 2 - 2b \rightarrow$ Solve for a and b.

$a = \frac{3h+3K+2}{2}$, $b = \frac{3h-3K-2}{4}$

$(a+2) = \left(\frac{3h+3K+6}{2}\right), b+2 = \left(\frac{3h-3K+6}{2}\right)$

$\Rightarrow \left(\frac{3h+3K+6}{2}\right)^2 + \left(\frac{3h-3K+6}{2}\right)^2 = 32$

$9(x + y + z)^2 + 9(x - y + z)^2 = 128$

$18[x^2 + y^2 + 4x + 4] = 128 \Rightarrow x^2 + y^2 + 4x - \frac{28}{9} = 0$

7. Evaluate

$$\lim_{x\to\infty}\left(\frac{2x^7+3x-5}{3x^7+5x-2}\right)^x \frac{(3x-1)^{\frac{x}{2}}}{\left(\sqrt{3x+2}\right)^x}$$

(1) $\frac{2}{3\sqrt{e}}$ (2) $\frac{3}{\sqrt{e}}$

(3) $\frac{2}{5\sqrt{e}}$ (4) $\frac{2}{\sqrt{e}}$

Answer (1)

Sol. $\lim\limits_{x\to\infty} \underbrace{\left[\frac{2x^7+3x-5}{3x^7+5x-2}\right]}_{M} \underbrace{\left(\frac{(3x-1)^{\frac{x}{2}}}{\left(\sqrt{3x+2}\right)^x}\right)}_{N}$

$$\lim_{x\to\infty} M = \lim_{x\to\infty}\left[\frac{2+\frac{3}{x^6}-\frac{5}{x^7}}{3+\frac{5}{x^6}-\frac{2}{x^7}}\right] = \frac{2}{3}$$

$$\lim_{x\to\infty} N = \lim_{x\to\infty}\left(\frac{3x-1}{3x+2}\right)^{\frac{x}{2}} = (1^\infty \text{ form})$$

$$= e^L$$

$$L = \lim_{x\to\infty}\left(\frac{3x-1}{3x+2}-1\right)\frac{x}{2}$$

$$= \lim_{x\to\infty} -\frac{3x}{2(3x+2)} = \lim_{x\to\infty} -\frac{3}{2\left(3+\frac{2}{x}\right)} = -\frac{3}{6} = -\frac{1}{2}$$

$$\therefore \quad \lim_{x\to\infty} N = e^{-\frac{1}{2}}$$

$$\therefore \quad \lim_{x\to\infty} MN = \frac{2}{3}e^{-\frac{1}{2}}$$

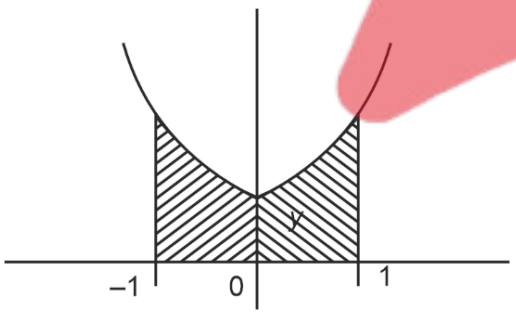
8. Let S be the region consisting of points (x, y) such that $-1 \le x \le 1$ and $0 \le y \le a + e^{|x|} - e^{-|x|}$. If area of bounded region is $\frac{2(e^2+8e+1)}{e}$ square units, then a is equal to

(1) 8 (2) 10

(3) 7 (4) 5

Answer (2)

Sol. $0 \le y \le a + e^{|x|} - e^{-|x|}$



$$\text{Area} = 2\int_0^1 \left(a + e^{|x|} - e^{-|x|}\right)dx$$

$$= 2\int_0^1 \left(a + e^{x} - e^{-x}\right)dx$$

$$= 2(ax + e^x + e^{-x})\Big|_0^1$$

$$= 2\,(a + e + e^{-1}) - 2\,(e^0 + e^{-0})$$

$$= 2\left(a + e + \frac{1}{e}\right) - 2(2)$$

$$= 2e + \frac{2}{e} + 2a - 4 = 2\left(e + \frac{1}{e} + (a-2)\right)$$

$$= \frac{2(e^2 + (a-2)(e+1)}{e}$$

Comparing, $a - 2 = 8 \Rightarrow a = 10$

9. If z is a complex number such that $|z| = 1$ and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$, then the number of complex number z is

(1) 8 (2) 4

(3) 2 (4) Zero

Answer (1)

Sol. $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$

$$\Rightarrow \left|z^2 + (\bar{z})\right| = 1$$

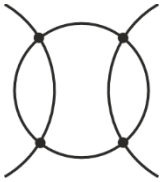
$$Z = x + iy$$

$$\Rightarrow \left|Z^2 + (\bar{Z})^2\right| = \left|2(x^2 - y^2)\right|$$

$$\Rightarrow x^2 - y^2 = \pm\frac{1}{2}$$

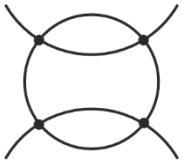
and $x^2 + y^2 = 1$

$x^2 - y^2 = \frac{1}{2}$ and $x^2 - y^2 = 1$



4 sol.

$x^2 - y^2 = \frac{-1}{2}, x^2 + y^2 = 1$



4 sol.

$\Rightarrow$ Total 8 sol.

10. If $A\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$, $A\begin{bmatrix}4\\1\\3\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $A\begin{bmatrix}2\\1\\2\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$. Then a_{23} equals to

(1) 3 (2) –1

(3) 2 (4) –2

Answer (2)

Sol. Let $A = \begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$

$\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$

$\Rightarrow$ $b = 0$ $e = 0$ $h = 1$

$\begin{bmatrix}a & 0 & c\\ d & 0 & f\\ g & 1 & i\end{bmatrix}\begin{bmatrix}4\\1\\3\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$

$4a + 3c = 0$

$4d + 3f = 1$ …(1)

$4g + p + 3i = 0$

$\begin{bmatrix}a & 0 & c\\ d & 0 & f\\ g & 1 & i\end{bmatrix}\begin{bmatrix}2\\1\\2\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$

$2a + 2c = 1$

$2d + 2f = 0$ …(2)

$2g + 1 + 2f = 0$

Solving (1) and (2)

$d = 1$ $f = -1$

$\Rightarrow$ $a_{23} = -1$

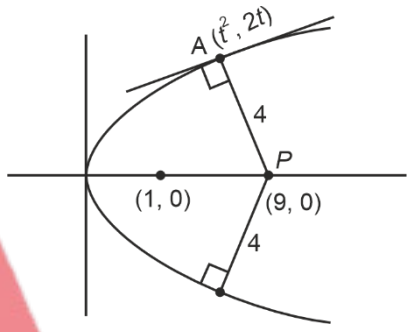
11. Let $(a, 0)$ be a point such that its shortest distance from the parabola $y^2 = 4x$ is 4. Then the equation of circle passing through $(a, 0)$ and focus of the parabola having centre on the axis of parabola is

(1) $x^2 + y^2 - 6x + 8 = 0$ (2) $x^2 + y^2 + 6x + 5 = 0$

(3) $x^2 + y^2 - 6x + 5 = 0$ (4) $x^2 + y^2 - 9x + 5 = 0$

Answer (3)

Sol.



The shortest distance lie along the normal.

Slope of normal at A is $(-t)$

$\Rightarrow$ $(-t) = \frac{2t - 0}{(t^2 - a)}$

$\Rightarrow$ $t = 0,\ t^2 - a = -2$

$t^2 = (a - 2)$

$(t^2 - a)^2 + (2t)^2 = 16$

$\Rightarrow$ $4 + 4t^2 = 16 \Rightarrow t^2 = 3,\ a = 5$

Equation of circle passing through (1, 0) and (5, 0) and having centre on the axis will be diametric form of circle.

$(x - 1)(x - 5) + (y - 0)(y - 0) = 0$

$x^2 + y^2 - 6x + 5 = 0$

12. If 10^{th} and 12^{th} terms of an arithmetic progression are roots of equation $3x^2 - px + q = 0$ and common difference of the arithmetic progression is $\frac{3}{2}$. Also, the sum of first 11 terms of the arithmetic progression is 88, then $q - 2p$ is

(1) 625 (2) 474

(3) 729 (4) 476

Answer (2)

Sol. $\Rightarrow \quad S_{11} = \frac{11}{2}(2a + (11-1)d) = 88$

$\Rightarrow \quad (a + 5d) = 8$

$a = 8 - 5\left(\frac{3}{2}\right) = \frac{1}{2}$

$T_{10} = a + 9d = \frac{1}{2} + 9\left(\frac{3}{2}\right) = \frac{1+27}{2} = 14$

$T_{11} = a + 10d = \frac{1}{2} + 10\left(\frac{3}{2}\right) = \frac{31}{2}$

$\Rightarrow$ Sum of roots $= 14 + \frac{31}{2} = \frac{P}{3}$

Product of roots $= (14)\frac{(31)}{2} = \frac{q}{3}$

$\frac{p}{3} = \frac{59}{2}, \quad q = (7.31)\cdot 3$

$P = \frac{59 \times 3}{2}, \quad q = 651$

$\Rightarrow \quad q - 2p = 651 - 59 \times 3 = 474$

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. There are 5 boys and 4 girls. The sum of number of ways in which they can be seated such that all boys sit together and number of ways such that no boys sit together is equal to

Answer (17280)

Sol. All boys sit together

$\boxed{B_1B_2B_3B_4B_5}\ G_1G_2G_3G_4$

= 5! 5!

no boys sit together

$\uparrow G_1 \uparrow G_2 \uparrow G_3 \uparrow G_4 \uparrow$

$4! \cdot 5!$

$\therefore$ sum = $5! \cdot 5! + 5! \cdot 4!$

$= 5!\,(4! + 5!)$

$= 5! \cdot 4!\,(1 + 5) = 6 \cdot 4!\,5! = 17280$

22. Let $f(x) = 6 + 16\cos\left(\frac{\pi}{3} - x\right)\cos\left(\frac{\pi}{3} + x\right)\cos x\ \sin 3x$ $\cos 6x$ if range of $f(x)$ is $[\alpha, \beta]$ then distance of (α, β) from $3x + 4y + 12 = 0$

Answer (11)

Sol. $f(x) = 6 + 16 \times \frac{1}{4}\cos 3x \sin 3x \cos 6x$

$= 6 + 2\sin 6x \cos 6x$

$= 6 + \sin 12x$

$R_{f(x)} \in [5, 7]$

Now distance $= \left|\frac{15 + 28 + 12}{\sqrt{3^2 + 4^2}}\right| = \frac{55}{5} = 11$

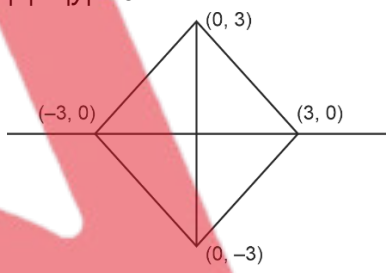
23. $A = \{(x, y) \,||\, x + y \,| \geq 3\}$;

$B = \{(x, y) \,||\, x \,| + |\, y \,| \leq 3\}$

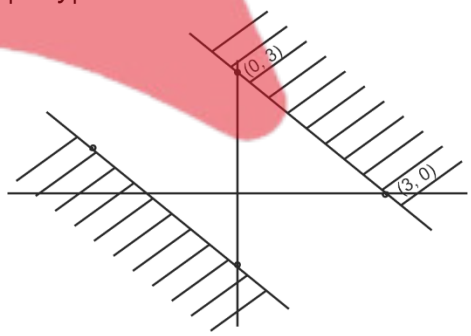
Let $C = A \cap B$. Find the sum of $(x, y) \forall\ x, y \in C$.

Answer (0)

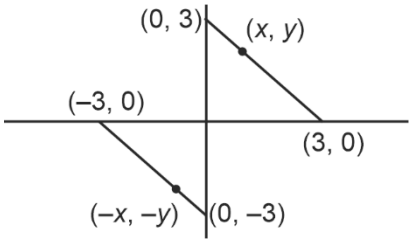
Sol. $|x| + |y| \leq 3$



$|x + y| \geq 3$



$A \geq B =$



$\therefore$ Sum = 0

24. If system of linear equations

$x + y + z = 6$

$x + 2y + 5z = 9$

$x + 5y + \lambda z = \mu$

has no solutions. Then value of λ equals to

Answer (17)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 5 & \lambda \end{vmatrix} = 0$

$\lambda - 17 = 0$

$\Rightarrow \lambda = 17$

For no solution $\Delta = 0$ at least one of $\Delta_1, \Delta_2, \Delta_3 \neq 0$

$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 9 & 2 & 5 \\ \mu & 5 & 17 \end{vmatrix} \neq 0$

$\Rightarrow 3\mu - 54 \neq 0$

$\mu \neq 18$

$\Rightarrow$ for $\mu \neq 18$ and $\lambda = 17$ given system of equations have no solution.

25. Let $\int x^3 \sin x dx = g(x) + C$, where $g(0) = 0$. If $8\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \alpha\pi^3 + \beta\pi^2 + \gamma$, where α, β and γ are integers, then the value of $\alpha + \beta + \gamma$ is

Answer (55)

Sol. $I = \int x^3 \sin x dx = x^3(-\cos x) - \int 3x^2(-\cos x)dx$

$= -x^3 \cos x + 3\left[x^2(\sin x) - \int 2x \sin x \, dx\right]$

$= -x^3 \cos x + 3x^2 \sin x - 6\left[x(-\cos x) - \int(-\cos x)dx\right]$

$= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + c$

$\because g(0) = 0 \Rightarrow g(x) = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x$

$= g\left(\frac{\pi}{2}\right) = \frac{3\pi^2}{4} - 6, \quad g'\left(\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\right)^3 \sin\left(\frac{\pi}{2}\right) = \frac{\pi^3}{8}$

$\Rightarrow 8\left[g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right] = 6\pi^2 - 48 + \pi^3 = \pi^3 + 6\pi^2 - 48$

$\Rightarrow \alpha = 1, \ \beta = 6, \ \gamma = -48$

❑ ❑ ❑