Arithmetic Progression:

- An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term a. The fixed number d is called its common difference.
- The general form of an AP is $a, a\ +\ d, a\ +\ 2d, a\ +\ 3d, \ldots$
- In the list of numbers a_1, a_2, a_3 ,... if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$ give the same value, i.e., if $a_{k+1} - a_k$ is the same for different values of k, then the given list of numbers is an AP.
- The nth term an (or the general term) of an AP is $a_n = a + (n-1)d$, where a is the first term and d is the common difference. Note that $a_1 = a$.
- The sum S_n of the first n terms of an AP is given by

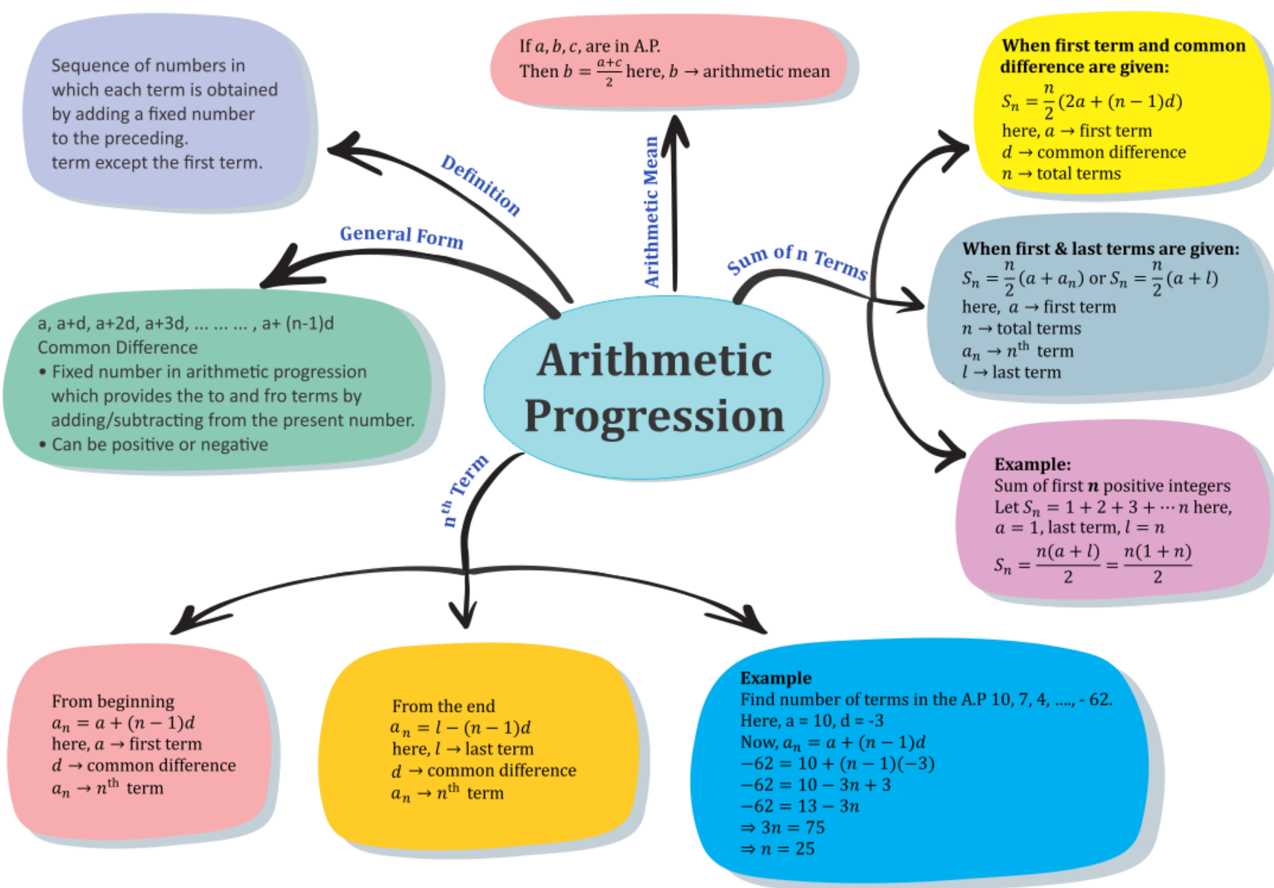
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

- If l is the last term of an AP of n terms, then the sum of all the terms can also be given by

$$S_n = \frac{n}{2}[a + 1]$$

- Sometimes S_n is also denoted by S.
- If S_n is the sum of the first n terms of an AP, then its nth term a_n is given by

$$a_n = S_n - S_{n-1}$$



Q1. Write the n^{th} term of the A.P. $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}$, ….

Sol. Given AP is

$\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, \dots \dots \dots \dots$

We have,

$a = \frac{1}{m}$

Now,

$d = \frac{1+m}{m} - \frac{1}{m} = \frac{1+m-1}{m} = 1$

We know that

$a_n = a + (n-1)d$

$\therefore a_n = \frac{1}{m} + (n-1)1 = \frac{1}{m} + n - 1$

Hence,

$a_n = \frac{1+(n-1)m}{m}$

Q2. For what value of k will $k+9, 2k-1$ and $2k+7$ are the consecutive terms of an A.P.?

Sol. We have

Consecutive terms of an AP are

k + 9, 2k – 1, 2k + 7

Then,

$\Rightarrow (k+9) + (2k+7) = 2(2k-1)$

{since if a, b, c are in AP, then $a + c = 2b$}

$\Rightarrow k + 9 + 2k + 7 = 4k - 2$

$\Rightarrow 3k + 16 = 4k - 2$

$\Rightarrow 16 + 2 = 4k - 3k$

$\Rightarrow k = 18$

Q3. If S_n, the sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$. Find the n^{th} term.

Sol. Given

$S_n = 3n^2 - 4n$

We have

$a_1 = S_1 = 3(1)^2 - 4(1) = 3 - 4 = -1$

$a_2 = S_2 - S_1$

$= [3(2)^2 - 4(2)] - (-1) = 12 - 8 + 1 = 5$

$\therefore d = a_2 - a_1 = 5 - (-1) = 6$

Hence,

$a_n = -1 + (n-1) \times 6 = 6n - 7$

Q4. The sum of the first 7 terms of an A.P. is 63 and that of its next 7 terms is 161. Find the A.P.

Sol. Given

$S_7 = 63$

We have,

$S_n = \frac{n}{2}[2a + (n-1)d]$

So,

$S_7 = \frac{7}{2}[2a + 6d] = 63$

Or

$2a + 6d = 18$ ... (i)

Now, sum of 14 terms is:

$S_{14} = S_{first\ 7\ terms} + S_{next\ 7\ terms}$

$= 63 + 161 = 224$

$\therefore \frac{14}{2}[2a + 13d] = 224$

$\Rightarrow 2a + 13d = 32$ ... (ii)

On subtracting (i) from (ii), we get

$(2a + 13d) - (2a + 6d) = 32 - 18$

$\Rightarrow$ 7d = 14

$\Rightarrow$ d = 2

Putting the value of d in (i), we get

$2a + 6(2) = 18$

$2a = 18 - 12$

a = 3

Hence, the A.P. will be: 3, 5, 7, 9, … ……….

Q5. Show that $(a-b)^2, (a^2+b^2)$ and $(a+b)^2$ are in A.P.

Sol. Given

$(a-b)^2, (a^2+b^2)$ and $(a+b)^2$

Common difference,

$d_1 = (a^2+b^2) - (a-b)^2 = a^2 + b^2 - (a^2 + b^2 - 2ab)$

$= a^2 + b^2 - a^2 - b^2 + 2ab = 2ab$

and $d_2 = (a+b)^2 - (a^2+b^2)$

$= a^2 + b^2 + +2ab - a^2 - b^2$

$= 2ab$

Since, $d_1 = d_2$

Hence, $(a-b)^2, (a^2+b^2)$ and $(a+b)^2$ are in A.P.

Q6. How many terms of the A.P. $-6, \frac{-11}{2}, -5, -\frac{9}{2}$ … …. are needed to give their sum zero.

Sol. Given $a = -6$ and $d = -\frac{11}{2} - (-6) = \frac{1}{2}$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

Let sum of n terms be zero.

$\therefore S_n = 0$

or, $\frac{n}{2}\left[2 \times -6 + (n-1)\frac{1}{2}\right] = 0$

or, $\frac{n}{2}\left[-12 + \frac{n}{2} - \frac{1}{2}\right] = 0$

or, $\frac{n}{2}\left[\frac{n}{2} - \frac{25}{2}\right] = 0$

or, $n^2 - 25n = 0$

$n(n-25) = 0$

$n = 25$ as $n \neq 0$

Hence, required terms are 25.

Q7. In a certain A.P. 32^{th} term is twice the 12^{th} term. Prove that 70^{th} term is twice the 31^{st} term.

Sol. Let the 1^{st} term be a and common difference be 'd'.

According to the question, $a_{32} = 2a_{12}$

$\therefore a + 31d = 2(a + 11d)$

$a + 31d = 2a + 22d$

$a = 9d$

Again, $a_{70} = a + 69d$

$= 9d + 69d = 78d$

$\because a_{31} = a + 30d$

$= 9d + 3d = 39d$

Hence, $a_{70} = 2a_{31}$

Hence Proved.

Q8. The 8^{th} term of an A.P. is zero. Prove that its 38^{th} term is triple of its 18^{th} term.

Sol. Given, $a_8 = 0$ or, $a + 7d = 0$ or, $a = -7d$

or, $a_{38} = a + 37d$

or, $a_{38} = -7d + 37d = 30d$

And, $a_{18} = a + 17d$

$= -7d + 17d = 10d$

or, $a_{38} = 30d = 3 \times 10d = 3 \times a_{18}$

$\therefore a_{38} = 3a_{18}$.

Hence Proved.

Q9. Show that the sum of all terms of an A.P. whose first term is a, the second term is b and the last term is c is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$.

Sol. Given, first term, A = a

and second term = b

$\Rightarrow$ common difference, $d = b - a$

Last term, $l = c$

$\Rightarrow A + (n-1)d = c$

[By using, $l = a + (n-1)d$]

$\Rightarrow a + (n-1)d = c$

$\Rightarrow a + (n-1)(b-a) = c$

$\Rightarrow (a-b)(n-1) = c - a$

$\Rightarrow n - 1 = \frac{c-a}{b-a}$

$\Rightarrow n = \frac{c-a}{b-a} + 1 = \frac{c-a+b-a}{b-a}$

$\Rightarrow n = \frac{b+c-2a}{b-a}$

Now sum $= \frac{n}{2}[A + l] = \frac{(b+c-2a)}{2(b-a)}\ [a + c]$

$= \frac{(a+c)(b+c-2a)}{2(b-a)}$

Q10. For what value of n, are the n^{th} terms of two A.Ps 63, 65, 67, … and 3, 10, 17, … equal?

Sol. Let a, d and A, D be the 1st term and common different of the 2 A.P.s respectively.

Here,

$a = 63, d = 2$

A = 3, D = 7

Given, $A_n = A_n$

$\Rightarrow a + (n-1)d = A + (n-1)D$

$\Rightarrow 63 + (n-1)2 = 3 + (n-1)7$

$\Rightarrow 63 + 2n - 2 = 3 + 7n - 7$

$\Rightarrow 61 + 2n = 7n - 4$

$\Rightarrow 5n = 65$

$\Rightarrow n = 13$

To solve such more questions click on the link below:

https://drive.google.com/file/d/1hXNNA6gOtCOPEUyJ9x-vvPt6eeqPu5xO/view?usp=drive_link