Multiple Choice Questions:

Q1. Three alarm clocks ring their alarms at regular intervals of 20 min., 25 min. and 30 min. respectively. If they first beep together at 12 noon, at what time will they beep again for the first time

(a) 4 : 00 pm

(b) 4 : 30 pm

(c) 5 : 00 pm

(d) 5 : 30 pm

Sol. Time when they ring together = LCM (20, 25, 30)

By prime factorisation,

$20 = 2 \times 2 \times 5$

$25 = 5 \times 5$

$30 = 2 \times 3 \times 5$

LCM $(20, 25, 30) = 2 \times 2 \times 3 \times 5 \times 5 = 300$

Thus, 3 bells ring together after 300 minutes or 5 hours.

Since, they rang together first at 12 noon, then they ring together again at 5 pm.

Q2. If a and b are two coprime numbers, then a^3 and b^3 are

(a) Coprime

(b) Not coprime

(c) Even

(d) Odd

Sol. As a and b are co-prime then a^3 and b^3 are also co-prime.

We can understand above situation with the help of an example.

Let $a = 3$ and $b = 4$

$a^3 = 3^3 = 27$ and $b^3 = 4^3 = 64$

Clearly,

$\text{HCF}(a, b) = \text{HCF}(3, 4) = 1$

Then,

$\text{HCF}(a^3, b^3) = \text{HCF}(27, 64) = 1$

Q3. A quadratic polynomial, the product and sum of whose zeroes are 5 and 8 respectively is

(a) $\boldsymbol{k[x^2 - 8x + 5]}$

(b) $\boldsymbol{k[x^2 + 8x + 5]}$

(c) $\boldsymbol{k[x^2 - 5x + 8]}$

(d) $\boldsymbol{k[x^2 + 5x + 8]}$

Sol. For any quadratic polynomial, $ax^2 + bx + c$

Sum of zeroes $= \frac{-b}{a}$

$\Rightarrow 8 = \frac{-b}{a}$

$\Rightarrow \frac{8}{1} = \frac{-b}{a}$

or $b = -8k, a = 1k$

Also, product of zeroes $= \frac{c}{a}$
$\Rightarrow 5 = \frac{c}{a}$
$\Rightarrow \frac{5}{1} = \frac{c}{a}$
or $c = 5k, a = 1k$
Polynomial whose sum of zeroes and product of zeroes are given is
$ax^2 + bx + c$
$= kx^2 - 8kx + 5k$
$= k(x^2 - 8x + 5)$

Q4. The zeroes of the polynomial $x^2 - 3x - m(m + 3)$ are

(a) $m, m + 3$
(b) $-m, m + 3$
(c) $m, -(m + 3)$
(d) $-m, -(m + 3)$

Sol. Given polynomial is
$x^2 - 3x - m(m + 3)$
putting $x = -m$, we get
$= (-m)^2 - 3(-m) - m(m + 3)$
$= m^2 + 3m - m^2 - 3m = 0$
putting $x = m + 3$, we get
$(m + 3)^2 - 3(m + 3) - m(m + 3)$
$= (m + 3)[m + 3 - 3 - m]$
$= (m + 3)[0] = 0$
Hence, – m and m + 3 are the zeroes of given polynomial.

Q5. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10
(b) –10
(c) –7
(d) –2

Sol. Given polynomial is
$p(x) = x^2 + 3x + k$
$\because$ 2 is a zero of $p(x)$, then $p(2) = 0$
$\therefore (2)^2 + 3(2) + k = 0$
$\Rightarrow 4 + 6 + k = 0$
$\Rightarrow 10 + k = 0$
$\Rightarrow k = -10$

Very Short Answer Type Questions:

Q6. Find the sum of exponents of prime factors in the prime factorisation of 196.

Sol. Prime factors of $196 = 2^2 \times 7^2$

The sum of exponents of prime factors = 2 + 2 = 4.

Q7. If HCF (336, 54) = 6, find LCM (336, 54)

Sol. Since HCF × LCM = Product of numbers

$\Rightarrow 6 \times LCM = 336 \times 54$

$\Rightarrow \text{LCM} = \frac{336\times54}{6}$

$\Rightarrow \text{LCM} = 56 \times 54$

$\Rightarrow \text{LCM} = 3024$

Q8. If the sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.

Sol. Given quadratic polynomial is

$3x^2 - kx + 6$

Let the roots of the given quadratic equation be α and β

So, we have

$\Rightarrow \alpha + \beta = \frac{k}{3}$

$\Rightarrow 3 = \frac{k}{3}$

$\Rightarrow k = 9$

Q9. If α and β are the zeroes of the polynomial $f(x) = x^2 - 6x + k$, find the value of k, such that $\alpha^2 + \beta^2 = 40$.

Sol. Given quadratic polynomial is

$f(x) = x^2 - 6x + k$

We have

$\alpha + \beta = -\frac{b}{a}$

$= \frac{-(-6)}{1} = 6$

and $\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$

Given,

$\alpha^2 + \beta^2 = 40$

$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 40$

$\Rightarrow (6)^2 - 2k = 40$

$\Rightarrow 36 - 2k = 40$

$\Rightarrow -2k = 4$

$\therefore k = -2$

Q10. Find the value of k such that the polynomial $x^2 - (k+6)x + 2(2k-1)$ has sum of its zeroes equal to half of their product.

Sol. Given polynomial is

$x^2 - (k+6)x + 2(2k-1)$

Now,

Sum of zeroes $= k + 6$

Product of zeroes $= 2(2k-1)$

ATQ.

$k + 6 = \frac{1}{2} \times 2(2k-1)$

$\Rightarrow$ k = 7

Short Answer Type Questions:

Q11. Check whether 4^n can end with the digit 0 for any natural number n.

Sol. If the number 4^n for any natural number n, were to end with the digit-zero, then it would be divisible by 5 and 2.

That is, the prime factorization of 4^n would contain the prime 5 and 2. This is not possible because $4^n = (2)^{2n}$; so, the only prime in the factorization of 4^n is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic states that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with the digit zero.

Q12. Prove that $2-\sqrt{3}$ is irrational, given that $\sqrt{3}$ is irrational.

Sol. Let $2-\sqrt{3}$ be a rational number

We can find co-prime numbers a and b $(b \neq 0)$ such that

$2-\sqrt{3} = \frac{a}{b}$

$2-\frac{a}{b} = \sqrt{3}$

So, we get, $\frac{2b-a}{b} = \sqrt{3}$

Since a and b are integers, we get $\frac{2b-a}{b} = \sqrt{3}$is irrational and so $\sqrt{3}$ is rational. But $\sqrt{3}$ is an irrational number.

But rational number cannot be equal to an irrational number.

Which contradicts our statement.

Therefore $2-\sqrt{3}$ is irrational.

Q13. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

Sol. Given quadratic equation is

$x^2 - 4\sqrt{3x} + 3 = 0$

If α and β are the zeroes of $x^2 - 4\sqrt{3}x + 3$

Then, $\alpha + \beta = -\frac{b}{a}$

$\Rightarrow \alpha + \beta = -\frac{(-4\sqrt{3})}{1}$

$\Rightarrow \alpha + \beta = 4\sqrt{3}$

and $\alpha\beta = \frac{c}{a}$

$\alpha\beta = \frac{3}{1}$

$\Rightarrow \quad \alpha\beta = 3$

$\therefore \alpha + \beta - \alpha\beta = 4\sqrt{3} - 3.$

Q14. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

Sol. Given polynomial is

$p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3} = \frac{1}{3}(21y^2 - 11y - 2)$

$= \frac{1}{3}(21y^2 - 14y + 3y - 2)$

$= \frac{1}{3}[7y(3y-2) + 1(3y-1)]$

$= \frac{1}{3}[(7y+1)(3y-2)]$

$\therefore$ Zeroes are $\frac{2}{3}, \frac{-1}{7}$

Sum of zeroes $= \frac{2}{3} - \frac{1}{7} = \frac{11}{21}$

$\frac{-b}{a} = \frac{11}{21} \{\therefore \text{sum of zeroes} = \frac{-b}{a}\}$

Product of zeroes $= \left(\frac{2}{3}\right)\left(-\frac{1}{7}\right) = -\frac{2}{21}$

$\frac{c}{a} = -\frac{2}{3}\left(\frac{1}{7}\right) = -\frac{2}{21} \left\{Product\ of\ zeroes = \frac{c}{a}\right\}$

Q15. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Sol. If α and β are the zeroes of $2x^2 - 3x + 1$.

then

$\alpha + \beta = \frac{-b}{a}$

$\Rightarrow \alpha + \beta = \frac{3}{2}$

and $\alpha\beta = \frac{c}{a}$

$\Rightarrow \alpha\beta = \frac{1}{2}$

New quadratic polynomial whose zeroes are 3α and 3β is :

x^2– (Sum of the roots) x + Product of the roots

$= x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$

$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$

$= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$

$= x^2 - \frac{9}{2}x + \frac{9}{2}$

$= \frac{1}{2}(2x^2 - 9x + 9)$

Hence, required quadratic polynomial is $\frac{1}{2}(2x^2 - 9x + 9)$

Long Answer Type Questions:

Q16. Show that the square of an odd positive integer can be of the form $6q + 1$ or $6q + 3$ for some integer q.

Sol. We know that any positive integer can be of the form $6m, 6m + 1, 6m + 2, 6m + 3, 6m + 4$ or $6m + 5$, for some integer m.

Thus, an odd positive integer can be of the form $6m + 1, 6m + 3, or\ 6m + 5$ Thus we have:

$(6m + 1)^2 = 36m^2 + 12m + 1 = 6(6m^2 + 2m) + 1 = 6q + 1, q$ is an integer

$(6m + 3)^2 = 36m^2 + 36m + 9 = 6(6m^2 + 6m + 1) + 3 = 6q + 3, q$ is an integer

$(6m + 5)^2 = 36m^2 + 60m + 25 = 6(6m^2 + 10m + 4) + 1 = 6q + 1, q$ is an integer.

Thus, the square of an odd positive integer can be of the form $6q + 1$ or $6q + 3$.

Q17. Prove that $\sqrt{7} + \sqrt{11}$ is an irrational number.

Sol. Let us assume to the contrary, $\sqrt{7} + \sqrt{11}$ be a rational number $\frac{a}{b}$ (a, b are integers, $b \neq 0$)

$\sqrt{7} + \sqrt{11} = \frac{a}{b}$

On squaring both the sides:

$7 + 11 + 2\sqrt{77} = \frac{a^2}{b^2}$

$\sqrt{77} = \frac{1}{2}\left(\frac{a^2}{b^2} - 18\right)$

$\sqrt{77} = \frac{a^2 - 18b^2}{2b^2}$

As a and b are integers.

$\frac{a^2 - 18b^2}{2b^2}$ is a rational number.

$\Rightarrow \sqrt{77}$ is a rational number.

But this contradicts the fact that $\sqrt{77}$ is irrational. Our assumption is wrong.

Hence, $\sqrt{7} + \sqrt{11}$ is an irrational number.

Q18. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k.

Sol. Given, $p(x) = 2x^2 + 5x + k$

Then, sum of zeros = $-\frac{coefficient\ of\ x}{coefficient\ of x^2}$

$\alpha + \beta = -\frac{5}{2}$

And product of zeroes = $\frac{constant\ term}{coefficient\ of\ x^2}$

$\alpha\beta = \frac{k}{2}$

According to the equation,

$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

Or $(\alpha + \beta)^2 - 2\alpha\beta + \alpha\beta = \frac{21}{4}$

$\Rightarrow \left(-\frac{5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$

$\Rightarrow \frac{k}{2} = \frac{25}{4} - \frac{21}{4} = \frac{4}{4} = 1$

Hence, $k = 2$.

Q19. Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and $-\frac{3}{2}$, respectively. Also, find its zeroes.

Sol. A quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ *and* $-\frac{3}{2}$ is

$x^2 - \sqrt{2}x - \frac{3}{2}$

$x^2 - \sqrt{2}x - \frac{3}{2} = \frac{1}{2}[2x^2 - 2\sqrt{2}x - 3]$

$= \frac{1}{2}\left[2x^2 + \sqrt{2}x - 3\sqrt{2}x - 3\right]$

$= \frac{1}{2}\left[\sqrt{2}x\left(\sqrt{2}x + 1\right) - 3\left(\sqrt{2}x + 1\right)\right]$

$= \frac{1}{2}\left[\sqrt{2}x + 1\right][\sqrt{2}x - 3]$

Hence, the zeros are $-\frac{1}{\sqrt{2}}$ and $\frac{3}{\sqrt{2}}$.

Q20. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $\boldsymbol{f(x) = ax^2 + bx + c, a \neq 0, c \neq 0}$.

Sol. Let α and β be the zeroes of the given polynomial $ax^2 + bx + c$

$\therefore \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now,

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{-b}{c}$

$\frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{a}{c}$

So, required quadratic polynomial is

$x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c}$

$p(x) = \frac{1}{c}(cx^2 + bx + a)$