General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* **38** *questions.* ***All*** *questions are* ***compulsory.***

(ii) *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E.***

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*

(v) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are short answer (SA) type questio[ns] carrying* ***3*** *marks each.*

(vi) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are long answer (LA) type que[stions] carrying* ***5*** *marks each.*

(vii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is* ***not*** *allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. Sum of two skew-symmetric matrices of same order is always a/an :

(A) skew-symmetric matrix

(B) symmetric matrix

(C) null matrix

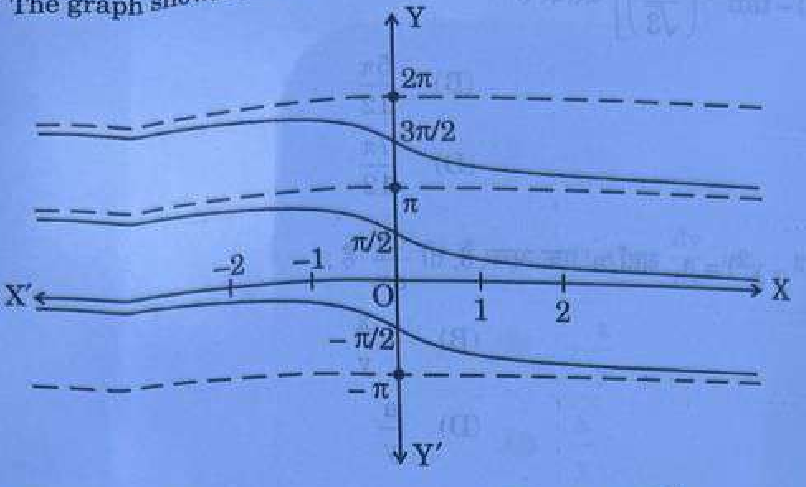
(D) identity matrix

2. If $A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 0 & 5 \\ -8 & -5 & 0 \end{bmatrix}$, then A is a :

(A) null matrix (B) symmetric matrix

(C) skew-symmetric matrix (D) diagonal matrix

3. The graph shown below depicts :



(A) $y = \cot x$

(B) $y = \cot^{-1} x$

(C) $y = \tan x$

(D) $y = \tan^{-1} x$

4. Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is :

(A) $n \times n$

(B) $n \times m$

(C) $m \times m$

(D) $m \times n$

5. If $f(x) = \begin{cases} \dfrac{\log(1 + ax) + \log(1 - bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$

is continuous at $x = 0$, then the value of k is :

(A) a

(B) a + b

(C) a – b

(D) b

6. If $y = a \cos(\log x) + b \sin(\log x)$, then $x^2y_2 + xy_1$ is :

(A) $\cot(\log x)$

(B) y

(C) – y

(D) $\tan(\log x)$

P.T.O.

7. $\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to :

(A) $\frac{11\pi}{12}$

(B) $\frac{5\pi}{12}$

(C) $-\frac{5\pi}{12}$

(D) $\frac{7\pi}{12}$

8. If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\frac{dy}{dx}$ is :

(A) $\frac{x}{y}$

(B) $-\frac{x}{y}$

(C) $\frac{a}{x}$

(D) $\frac{a}{y}$

9. Let $f(x) = x^2$, $x \in R$. Then, which of the following statements is ***incorrect*** ?

(A) Minimum value of f does not exist.

(B) There is no point of maximum value of f in R.

(C) f is continuous at $x = 0$.

(D) f is differentiable at $x = 0$.

10. $\int \frac{x+5}{(x+6)^2} e^x \, dx$ is equal to :

(A) $\log(x+6) + C$

(B) $e^x + C$

(C) $\frac{e^x}{x+6} + C$

(D) $\frac{-1}{(x+6)^2} + C$

11. Let $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, $f(1) = 0$. Then, $f(x)$ is :

(A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$

(B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$

(C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$

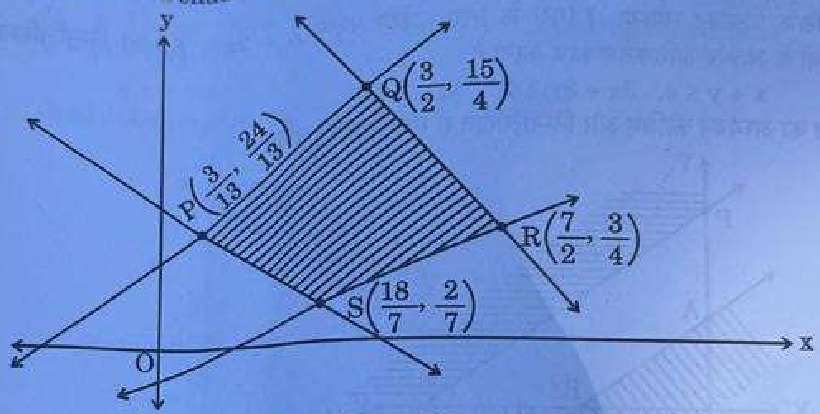
(D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$

12. The order and degree of the differential equation

$$\frac{d^2y}{dx^2} + 4\left(\frac{dy}{dx}\right) = x \log\left(\frac{d^2y}{dx^2}\right)$$ are respectively :

(A) 0, 3
(B) 2, 1
(C) 2, not defined
(D) 1, not defined

13. For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.



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$$P \equiv \left(\frac{3}{13}, \frac{24}{13}\right), Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right), R \equiv \left(\frac{7}{2}, \frac{3}{4}\right), S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$$

Which of the following statements is correct ?

(A) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$

(B) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$

(C) (Value of Z at P) > (Value of Z at Q)

(D) (Value of Z at Q) < (Value of Z at R)

14. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is :

(A) $\frac{3}{2}$ sq units
(B) $\frac{2}{3}$ sq units
(C) 3 sq units
(D) $\frac{4}{3}$ sq units

15. Let $|\vec{a}| = 5$ and $-2 \le \lambda \le 1$. Then, the range of $|\lambda \vec{a}|$ is :

(A) $[5, 10]$ (B) $[-2, 5]$

(C) $[-2, 1]$ (D) $[-10, 5]$

16. The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is :

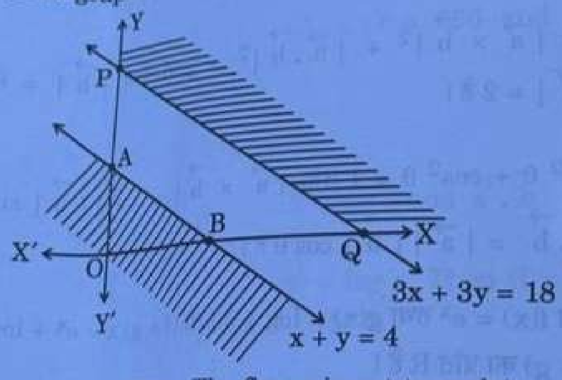
(A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$

(C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$

17. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints :

$x + y \le 4,\ 3x + 3y \ge 18,\ x, y \ge 0$

Study the graph and select the correct option.



(Note : The figure is not to scale)

The solution of the given LPP :

(A) lies in the shaded unbounded region.

(B) lies in Δ AOB.

(C) does not exist.

(D) lies in the combined region of Δ AOB and unbounded shaded region.

18. Chances that three persons A, B, and C go to the market are 30%, 60% and 50% respectively. The probability that at least one will go to the market is :

(A) $\frac{14}{10}$ (B) $\frac{43}{50}$

(C) $\frac{9}{100}$ (D) $\frac{7}{50}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 256$ and $|\vec{b}| = 8$, then $|\vec{a}| = 2$.

Reason (R) : $\sin^2\theta + \cos^2\theta = 1$ and $|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin\theta$ and $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$.

20. *Assertion (A) :* Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)\, x = e^x + \log x$ where domain of $(f + g)$ is R.

Reason (R) : $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$.

OR

(b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.

P.T.O.

22. (a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA.

OR

(b) Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.

23. Determine those values of x for which $f(x) = \frac{2}{x} - 5, x \neq 0$ is increasing or decreasing.

24. Find the domain of $f(x) = \sin^{-1}(-x^2)$.

25. Find the value of λ if the following lines are perpendicular to each other :

$$l_1 : \frac{1-x}{-3} = \frac{3y-2}{2\lambda} = \frac{z-3}{3}$$

$$l_2 : \frac{x-1}{3\lambda} = \frac{1-y}{1} = \frac{2z-5}{3}$$

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. If $A = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 3 & 4 \\ 0 & 5 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, are three matrices, then find ABC.

27. Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints

$$2x + y \geq 1000$$

$$x + 2y \geq 800$$

$$x, y \geq 0.$$

Draw a neat graph of the feasible region and find the minimum value of Z.

28. (a) Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

OR

(b) Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.

29. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

OR

(b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.

30. (a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

OR

(b) Show that the function $f : N \to N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, & \text{if n is even} \\ n+1, & \text{if n is odd} \end{cases}$ is a bijection.

31. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed three times, find the probability distribution of number of tails. Hence, find the mean of the distribution.

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Solve the differential equation : $x^2y\,dx - (x^3 + y^3)\,dy = 0$.

OR

(b) Solve the differential equation $(1 + x^2)\dfrac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.

33. Use integration to find the area of the region enclosed by curve $y = -x^2$ and the straight lines $x = -3$, $x = 2$ and $y = 0$. Sketch a rough figure to illustrate the bounded region.

34. (a) Find :

$$\int \frac{x^2+1}{(x-1)^2\,(x+3)}\,dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\,dx$$

35. Find the foot of the perpendicular drawn from point $(2, -1, 5)$ to the line $\dfrac{x-11}{10} = \dfrac{y+2}{-4} = \dfrac{z+8}{-11}$. Also, find the length of the perpendicular.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

A person buys a smartphone from this shop.

(i) Find the probability that it was defective. 2

(ii) What is the probability that this defective smartphone was manufactured by company B ? 2

Case Study – 2

37. Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.

Based upon the above information, answer the following questions :

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form $AX = B$. *1*

(ii) Find $|A|$ and confirm if it is possible to find A^{-1}. *1*

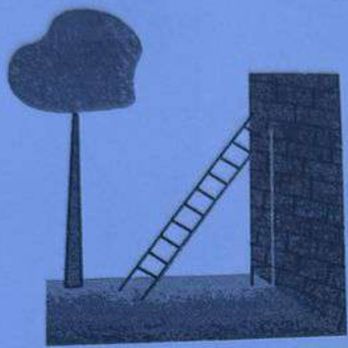
(iii) (a) Find A^{-1}, if possible, and write the formula to find X. 2

OR

(iii) (b) Find $A^2 - 8I$, where I is an identity matrix. 2

Case Study – 3

38.



A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions :

(i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. *1*

(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point. *1*

(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. 2

OR

(iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ? 2