S1. Ans. (c)

$g' = \frac{GM'}{R'^2} = \frac{GM}{10\left(\frac{R}{2}\right)^2}$

$= \frac{4}{10}\frac{GM}{R^2} = 0.4 \times 9.8$

$= 3.92\ m\,s^{-2}$.

S2. Ans. (d)

Apply energy conservation,

$U_i + K_i = U_f + K_f$

$\Rightarrow -\frac{GMm}{R} + K_i = -\frac{GMm}{3R} + \frac{1}{2}mv^2$

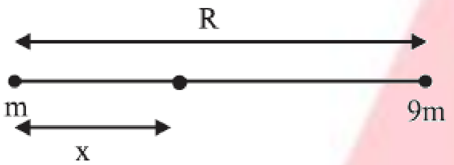
$\Rightarrow -\frac{GMm}{R} + K_i$

$= -\frac{GMm}{3R} + \frac{1}{2} \times m \times \frac{GM}{3R}$

$\Rightarrow K_i = -\frac{1}{6}\frac{GMm}{R} + \frac{GMm}{R}$

$K_i = \frac{5}{6}\frac{GMm}{R}$.

S3. Ans. (d)



Let the gravitational field is zero at a distance x from the mass m.

$\frac{Gm}{x^2} = \frac{G9m}{(R-x)^2}$

$\Rightarrow R - x = 3x$ or $x = \frac{R}{4}$

Gravitational potential at $\frac{R}{4}$

$= -\frac{Gm}{\frac{R}{4}} - \frac{G9m}{\frac{3R}{4}}$

$= -\frac{4Gm}{R} - \frac{12Gm}{R}$

$= -\frac{16Gm}{R}$

S4. Ans. (c)

Time period of satellite

$T = 2\pi\sqrt{\frac{R^3}{GM}}$

$= 2\pi\sqrt{\frac{R^3}{Gd\frac{4}{3}\pi R^3}}$

$\Rightarrow T = \sqrt{\frac{3\pi}{Gd}}$

S5. Ans. (a)

$I_g = \frac{F}{m}$

$= \frac{3}{60 \times 10^{-3}} = 50\ N/kg$

S6. Ans. (a)

Gravitational constant = $[M^{-1}L^3T^{-2}]$

Gravitational potential energy = $[ML^2T^{-2}]$

Gravitational potential = $[L^2T^{-2}]$

Gravitational intensity = $[LT^{-2}]$

S7. Ans. (c)

Hint: PE + KE = mgs

At given point

KE = 3PE

So, 4PE = mgs

H = s/4

KE = KE = $\frac{3mgs}{4} = \frac{1}{2}mV^2$

$V = \sqrt{\frac{3gs}{2}}$

S8. Ans. (c)

Hint: $Ve = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}} \times \frac{4}{3}\pi R3\rho$

$= \sqrt{\frac{8\pi G\rho}{3}}R$

$\Rightarrow V_e \infty R$

$\Rightarrow V_e/\upsilon = \frac{4R}{R} \Rightarrow V_e = 4v$

S9. Ans. (c)

Hint: $-\frac{GMn}{R} + \frac{1}{2}\ mk^2V_e^2 = -\frac{GMm}{R+r}$

$-\frac{GMn}{R} + \frac{1}{2}mk^2\frac{2GMm}{R} = -\frac{GMn}{R+r}$

$-\frac{1}{R} + \frac{k^2}{R} = -\frac{1}{R+r}$

$\frac{1}{R+r} = \frac{1}{R} - \frac{k^2}{R}$

$\frac{1}{R+r} = \frac{1-k^2}{R}$

$r = \frac{Rk^2}{1-k^2}$

S10. Ans. (a)

Hint: $w_s = mg_s = 72\ N$

$w_h = mg_h = \frac{mg_s}{\left(1+\frac{h}{R}\right)^2} = \frac{72N}{\left(1+\frac{\frac{R}{2}}{2}\right)^2} = \frac{\frac{72}{9}}{4}$

$W_h = 32\ N$

S11. Ans. (a)

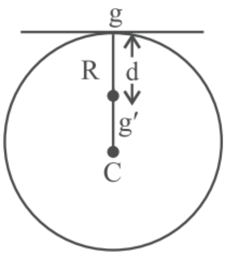
Hint: Inside the earth at depth d from the surface

$g_{eff} = g\left(1-\frac{d}{R}\right) \Rightarrow \frac{g}{n} = g\left(1-\frac{d}{R}\right)$

$\Rightarrow d = \frac{(n-1)R}{n}$

S12. Ans. (d)

Hint:



Acceleration due to gravity at a depth d from surface of earth

$g' = g\left(1-\frac{d}{R}\right)$ ....(1)

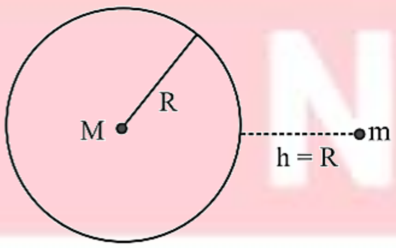
Here g = acceleration due to gravity at earth's surface Multiplying by mass 'm' on both sides of a.

$mg' = mg\left(1-\frac{d}{R}\right); \left(\because d = \frac{R}{2}\right)$

$= 200\left(1-\frac{R}{2R}\right) = \frac{200}{2} = 100\ N$

S13. Ans. (c)

Hint:



Initial potential energy

$U_i = \frac{-GMn}{R}$

Final potential energy at height h = R

$U_f = \frac{-GMn}{(R+R)}$

As work done = Change in PE

$\therefore W = U_f - U_i$

$= \frac{GMm}{2R} = \frac{gR^2 m}{2R} = \frac{mgR}{2} \quad (\because GM = gR^2)$

S14. Ans. (b)

Hint: As angular momentum is conserved mvr = constant so $v_A > v_B > v_C$

hence

$K_A > K_B > K_C$

S15. Ans. (d)

Hint: If universal constant becomes 10 times, then on earth surface

$g = \frac{GM}{R^2}$ $\quad\quad G' = 10\ G$

$g' = 10\frac{10GM}{R^2}$ $\quad\quad g' = 10g$

So g on earth surface will change

S16. Ans. (c)

Hint: Acceleration due to gravity at height 1 km above earth surface

$a_h = g\left[1-\frac{2R}{R}\right]$

$= g\left[1-\frac{2\times1}{R}\right]$

$= g\left[1-\frac{2}{R}\right]$

Acceleration due to gravity at depth 'd' below earth surface

$a_d = g\left[1-\frac{d}{R}\right]$

$a_h = a_d$

$g\left[1-\frac{2}{R}\right] = g\left[1-\frac{d}{R}\right]$

$d = 2\ km$

S17. Ans. (a)

Hint: Since two astronauts are floating in gravitational free space, the only force acting on the two astronauts is the gravitational pull of their masses

$F = \frac{Gm_1m_2}{r^2}$

which is attractive in nature. Hence they move towards each other.

S18. Ans. (b)

Hint: $g_d = g\left[1-\frac{d}{R}\right]$

$g_d = \frac{g}{4}$

$\frac{g}{4} = g\left[1-\frac{d}{R}\right]$

$\frac{d}{R} = \frac{3}{4}$ ....(1)

$g_h = g\left[\frac{R}{R+h}\right]^2$

$g_h = \frac{g}{4}$

$\frac{g}{4} = g\left[\frac{R}{R+h}\right]^2$

$\frac{h}{R} = 1$ ....(2)

From Eqs. (1) and (2), we get

$\frac{d}{h} = \frac{3}{4}$ $\qquad \frac{h}{d} = \frac{4}{3}$

S19. Ans. (d)

Hint: Satellite in a orbit of radius R

$TE = -\frac{GM_cm}{2R_E}$

When satellite in a orbit of radii $3R_E$

$TE_1 = -\frac{GM_cm}{2(3R_E)}$

When satellite in a orbit of radii $9R_E$

$TE_2 = \frac{-GM_cm}{2(9R_E)}$

$\Delta TE = TE_2 - TE_1$

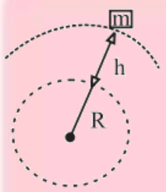
$= -\frac{GM_cm}{18R_E} - \left[-\frac{GM_cm}{6R_E}\right]$

$= \frac{GM_cm}{R_E}\left[\frac{1}{6} - \frac{1}{18}\right]$

$= \frac{GM_cm}{9R_E}$

S20. Ans. (d)

Hint:



$PE = -\frac{GMm}{R+h}$

$KE = \frac{1}{2}m\left[\sqrt{\frac{GM}{R+h}}\right]^2$

$KE = \frac{GMm}{2(R+h)}$

$TE = \frac{-GMm}{R+h} + \frac{GMn}{2[R+h]}$

$= \frac{-GMn\,R^2}{2(R+h)R^2}$ $\qquad \left[\because g = \frac{GM}{R^2}\right]$

$= \frac{-g_0mR^2}{2[R+h]}$

S21. Ans. (d)

Hint: $g = \left(\frac{GM_e}{R_e^3}\right)r$ for $0 < r \le R_e \Rightarrow g \propto r$

$g = \frac{GM_e}{r^2}$ for $r > R_e \Rightarrow g \propto \frac{1}{r^2}$

S22. Ans. (b)

Hint: g [in terms of mean density]

$= \frac{4\pi GdR}{3} \Rightarrow V_e = \sqrt{2gR}$

$V_e = \sqrt{2 \times \frac{4}{3}\pi GdR.R}$

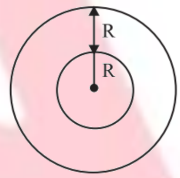
$V_e = \sqrt{\frac{8}{3}\pi GdR^2}$

$V_p = \sqrt{\frac{8}{3}\pi G2d(2R)^2} \because d_p = 2d, R_p = 2R$

$\frac{V_e}{V_p} = \frac{1}{\sqrt{8}} \Rightarrow \frac{1}{2\sqrt{2}}$

S23. Ans. (a)

Hint:



Gravitational Potential

$= \frac{-GM}{R+h} = -5.4 \times 10^7 \,J\,Kg^{-2}$ ....(1)

$g = \frac{GM}{(R+h)^2} = 6\ m/s^2$..........................(2)

$(1)/(2) = (R+h) = \frac{5.4\times10^7}{6} = 0.9 \times 10^7 m$

= 9000 km

H = 9000 km – 6400 km = 2600 km

S24. Ans. (a)

Hint: $T_p^2 = Kr^3$ Given ....(1)

$T_{planet} = \frac{2\pi r}{V_p}$ $\left[\because V_p = \sqrt{\frac{GM}{r}}\right]$

$= \frac{2\pi r.r^{\frac{1}{2}}}{\sqrt{GM}}$

$T_p = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{GM}}$ ....(2)

Squaring Eq. (2), we have

$T_p^2 = \frac{4\pi^2r^3}{GM}$

$Kr^3 = \frac{4\pi^2r^3}{GM} \Rightarrow K = \frac{4\pi^2}{GM}$

S25. Ans. (b)

Hint: For the satellite revolving around earth

$v_0 = \sqrt{\frac{GM_e}{(R_e+h)}} \quad \sqrt{\frac{GM_e}{R_e\left(+\frac{h}{R_e}\right)}} = \sqrt{\frac{gR}{1+\frac{h}{R_e}}}$

Substituting the values

$v_0 = \sqrt{60 \times 60^6}$ m/s

$v_0 = 7.76 \times 10^3$ m/s = 7.76 km/s

S26. Ans. (a)

Hint:

Acceleration due to earth to the satellite is centripetal, hence directed towards centre. Angular momentum conservation holds good for comparable masses but

$M_{earth} \gg M_{satelite}$

S27. Ans. (a)

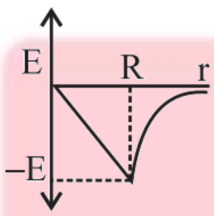
Hint: Gravitational field intensity

$\vec{E} = \frac{-GM}{R^2}$

For a point inside the earth $E = \frac{-GMr}{R^3}$

For a point inside the earth $E = \frac{-GM}{R^2}$

Where –ve sign indicates the attractive gravitational field



Accurate graph to show variation of E with r

S28. Ans. (a)

Hint: Escape velocity $(v_e) = \sqrt{\frac{2GM}{R}} = c =$ speed of light

$\Rightarrow R = \frac{2GM}{c^2} = \frac{2\times6.6\times10^{-11}\times5.98\times10^{24}}{(3\times10^8)^2}$ m

$= 10^{-2}$m

S29. Ans. (a)

Hint: $V = -G(2)\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots..\right]$

Because it forms geometric progression

$S_{GP} = \frac{r^n-1}{r-1}; \qquad r = \frac{1}{2}$

$\frac{\left(\frac{1}{2}\right)^n-1}{-1/2} = \frac{-1}{-1/2} = 2$

$\therefore V = -2G \times S_{GP}$

$= -2G \times 2 = -4G$

S30. Ans. (c)

Hint: Change in P.E. $= -\frac{GMm}{3R} - \left(-\frac{GMm}{R}\right)$

$= \frac{2}{3}\frac{GMm}{R} = \frac{2}{3}$ mgR