**Hall Ticket Number**

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**Q.B. No.** 100061

**Booklet Code : A**

# DL-313-MAT

**Marks : 100**

**Time : 120 Minutes**

**Paper-II**

Signature of the Candidate

Signature of the Invigilator

## INSTRUCTIONS TO THE CANDIDATE
**(Read the Instructions carefully before Answering)**

1. Separate Optical Mark Reader (OMR) Answer Sheet is supplied to you along with Question Paper Booklet. Please read and follow the instructions on the OMR Answer Sheet for marking the responses and the required data.
2. The candidate should ensure that the **Booklet Code printed on OMR Answer Sheet and Booklet Code supplied are same.**
3. **Immediately on opening the Question Paper Booklet by tearing off the paper seal, please check for *(i)* The same booklet code (A/B/C/D) on each page. *(ii)* Serial Number of the questions (1-100), *(iii)* The number of pages and *(iv)* Correct Printing.** In case of any defect, please report to the invigilator and ask for replacement of booklet with same code within five minutes from the commencement of the test.
4. Electronic gadgets like Cell Phone, Calculator, Watches and Mathematical/Log Tables are not permitted into the examination hall.
5. **There will be 1/4 negative mark for every wrong answer.** However, if the response to the question is left blank without answering, there will be no penalty of negative mark for that question.
6. Record your answer on the OMR answer sheet by using Blue/Black ball point pen to darken the appropriate circles of (1), (2), (3) or (4) corresponding to the concerned question number in the OMR answer sheet. Darkening of more than one circle against any question automatically gets invalidated and will be treated as wrong answer.
7. Change of an answer is **NOT** allowed.
8. Rough work should be done only in the space provided in the Question Paper Booklet.
9. **Return the OMR Answer Sheet and Question Paper Booklet to the invigilator before leaving the examination hall.** Failure to return the OMR sheet and Question Paper Booklet is liable for criminal action.

This Booklet consists of **21** Pages for **100** Questions +**2** pages of Rough Work +**1** Title Page i.e. Total **24** pages

1. Let S be a non-empty bounded set in **R**. Let $b < 0$ and let $bS = \{bs \mid s \in S\}$. Then which of the following is *true* ?

(1) Inf ($b$S) = $b$ Inf S, Sup ($b$S) = $b$ Sup S

(2) Inf ($b$S) = $b$ Inf S, Sup ($b$S) = $b$ Inf S

(3) Inf ($b$S) = $b$ Sup S, Sup ($bs$) = $b$ Inf S

(4) Inf ($b$S) = $b$ Sup S, Sup ($b$S) = $b$ Sup S

2. Which of the following is the statement of Bolzano-Weierstrass theorem ?

(1) A sequence of real numbers has a convergent subsequence

(2) A bounded sequence of real numbers has a convergent subsequence

(3) A sequence of real numbers has a monotone subsequence

(4) Any subsequence of a convergent sequence converges to the same limit

3. Let the sequence $(x_n)$ be defined by $x_1 = \sqrt{2}, x_{n+1} = \sqrt{2x_n}$ for every $n \subset \mathbf{N}$. The sequence $(x_n)$ is :

(1) convergent and converges to 3

(2) convergent and converges to 2

(3) divergent

(4) convergent and converges to 4

4. Consider $A : \sum_{n-1}^{\infty} ne^{-n^2}$ and $B : \sum_{n=1}^{\infty} \frac{1}{2^{\ln n}}$. Which of the following is *true* ?

(1) Both A and B converge

(2) A converges and B diverges

(3) A diverges and B converges

(4) Both A and B diverge

5. The set of all values of $x$ for which the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)}$ converges is :

(1) **R**

(2) $[-1, 1]$

(3) $[-1, 1)$

(4) $(-1, 1)$

6. Let S be a countable set. Which of the following statements is *false* ?

(1) There exists a surjection of **N** onto S

(2) Any $T \subseteq S$ is countable

(3) There does not exist an injection of S into **N**

(4) There exists an injection of S into **N**

7. Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined by $f(x) = \begin{cases} 1, \text{ if } x \text{ is rational} \\ 0, \text{ if } x \text{ is irrational} \end{cases}$.

Then

(1) $f$ is continuous at all rationals and discontinuous at $x = \sqrt{2}$

(2) $f$ is continuous at all irrationals and discontinuous at $x = 0$

(3) $f$ is not continuous at any point of **R**

(4) $f$ is continuous for all $x \in \mathbf{R}$

8. Which of the following is uniformly continuous ?

(1) $f : \{x \in \mathbf{R} \mid x > 0\} \to \mathbf{R},\ f(x) = \frac{1}{x}$

(2) $f : \mathbf{R} \to \mathbf{R},\ f(x) = x^2$

(3) $f : \left[\frac{1}{2}, 1\right] \to \mathbf{R},\ f(x) = \frac{1}{x}$

(4) $f : (0, 6] \to \mathbf{R},\ f(x) = \sin\left(\frac{1}{x}\right)$

9. Suppose that $f(x)$ is continuous and differentiable on $[-7, 0]$ and $f(-7) = -3$. If $f'(x) \leq 2$ for every $x \in [-7, 0]$, then the largest possible value for $f(0)$ is :

(1) 11 (2) 8

(3) 0 (4) 15

10. Let $A \subseteq \mathbf{R}$. Which of the following is *true* if $f : A \to \mathbf{R}$ is uniformly continuous ?

(1) If $(x_n)$ is a Cauchy sequence in A, then $(f(x_n))$ is a Cauchy Sequence in $\mathbf{R}$

(2) There exists a constant $k > 0$ such that $|f(x) - f(u)| \leq k|x - u|$ $\forall\ x, u \in A$

(3) $f$ need not be continuous on A.

(4) There exists an $\in > 0$ and two sequences $(x_n)$ and $(u_n)$ in A such that $\lim_{n\to\infty}(x_n - u_n) = 0$ and $|f(x_n) - f(u_n)| \geq \in \ \forall\ n \in \mathbf{N}$

11. Let $f(x) = x^2$ on $[0, 1]$ and $P = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$ be a partition of $[0, 1]$. Then the value of the lower Riemann sum, $L(f; P)$ is :

(1) $\frac{7}{16}$ (2) $\frac{7}{32}$

(3) $\frac{7}{33}$ (4) $\frac{7}{34}$

12. Let $f_n : [-1, 1] \to \mathbf{R}$ be given by $f_n(x) = x^n$, $\forall\ n \in N$. Then which of the following is *true* ?

(1) $(f_n)$ converges pointwise to $f$, where $f(x) = \begin{cases} 0 \text{ for } -1 \leq x < 1 \\ 1 \text{ for } x = 1 \end{cases}$

(2) $(f_n)$ converges uniformly to $f$, where $f(x) = \begin{cases} 0 \text{ for } -1 \leq x < 1 \\ 1 \text{ for } x = 1 \end{cases}$

(3) $(f_n)$ converges uniformly to $f$, where $f(x) = \begin{cases} 0 \text{ for } -1 < x < 1 \\ 1 \text{ for } x = 1 \end{cases}$

(4) $(f_n)$ converges pointwise to $f$, where $f(x) = \begin{cases} 0 \text{ for } -1 < x < 1 \\ 1 \text{ for } x = 1 \end{cases}$

13. Consider the metric $\rho$ on $\mathbf{R}^2$ defined as $\rho((x_1, y_1), (x_2, y_2)) = \max \{|x_1 - x_2|, |y_1 - y_2|\}$ for every $(x_1, y_1), (x_2, y_2) \in \mathbf{R}^2$

Under this metric $\rho$, the open ball of radius 1, centered at the point (0, 0) is the set of points $(x, y)$ such that

(1) $x^2 + y^2 < 1$

(2) $0 < x < 1$ and $0 < y < 1$

(3) $-1 < x < 1$ and $-1 < y < 1$

(4) $-1 < x + y < 1$

14. Among the following collections of subjects of $\mathbf{R}$, the collection that forms a topology on $\mathbf{R}$ is :

(1) $\{(a, b)/a, b \in \mathbf{R}\}$

(2) $\{(a, b)/a, b \in \mathbf{Q}\}$

(3) $\{U \subseteq \mathbf{R}/\text{either } U = \phi \text{ or } \mathbf{R} \setminus U \text{ is finite}\}$

(4) $\{U \subseteq \mathbf{R}/\text{either } U = \mathbf{R} \text{ or } \mathbf{R} \setminus U \text{ is infinite}\}$

15. Let X and Y be topological spaces and $f : X \to Y$ be a function. $f$ is a continuous function if and only if :

(1) image of every open set in X, under $f$ is open in Y

(2) for any bases $\mathbf{B}_x$ and $\mathbf{B}_y$ of X and Y respectively, $f^{-1}$ $(B) \in \mathbf{B}_x \forall B \in \mathbf{B}_y$

(3) $f(\bar{A}) \subseteq \overline{f(A)}$ for every $A \subset X$

(4) image of every closed set in X, under $f$ is closed in Y

16. Which of the following is a sufficient condition for a metric space, X to be a complete metric space ?

(1) Every Cauchy sequence has a convergent subsequence

(2) Every sequence has a monotonic subsequence

(3) Every Cauchy sequence is bounded

(4) Every convergent sequence is a Cauchy sequence

17. Consider the subsets A and B of $\mathbf{R}^2$ defined as :

$$A = \left\{\left(x, \sin\frac{1}{x}\right)/x \in (0, 1]\right\} \text{ and } B = A \cup \{(0, 0)\}$$

Under the standard topology on $\mathbf{R}^2$,

(1) A is compact and connected

(2) A is connected but not compact

(3) B is compact and connected

(4) B is compact but not connected

18. Which of the following is *not* a basis for any topology on **R** ?

(1) $\{[a, b]/a, b \in \mathbf{R} \text{ and } a \leq b\}$

(2) $\{(a, b)/a, b \in \mathbf{Q} \text{ and } a < b\}$

(3) $\{A \subset \mathbf{R}/A \text{ is countable}\}$

(4) $\{A \subset \mathbf{R}/A \text{ is uncountable}\}$

19. Let X be a subspace of the space of real numbers with standard topology and let $f : X \to \mathbf{R}$ be a continuous function. Which of the following implies that $f$ is uniformly continuous ?

(1) X is closed

(2) X is bounded

(3) X is compact

(4) For any sequence $(x_n)$ in X converging to a point $x$ in X, $(f(x_n))$ converges to $f(x)$

20. A metric space X is compact if any only if :

(1) every infinite subset of X has a limit point

(2) every class of subsets of X with finite intersection property has non-empty intersection

(3) X is closed and bounded

(4) X is a complete metric space

21. Let $a$ and $b$ be positive integers such that $a = \prod_{n=1}^{\infty} p_n^{a_n}$ and $b = \prod_{n=1}^{\infty} p_n^{b_n}$, where each $a_n$ and $b_n$ is a non-negative integer and $\{p_n / n \in \mathbf{N}\}$ is the set of all prime numbers. Then the g.c.d. of $a$ and $b$ is $\prod_{n=1}^{\infty} p_n^{c_n}$, where $c_n$ =

(1) $a_n + b_n$ (2) max $\{a_n, b_n\}$

(3) min $\{a_n, b_n\}$ (4) g.c.d. of $a_n$ and $b_n$

22. If $p$ is an odd prime number and $p \mid 38^{2019}$, then $p$ =

(1) 19

(2) 2019

(3) either 19 or 2019

(4) a prime number other than 19 and 2019

23. If $m$ and $n$ are positive integers such that $m$ is odd and $n$ is a power of 2, then the g.c.d. of $m + 2n$ and $m - 2n$ is

(1) 1

(2) $2^k$, for some $k \in \mathbf{N}$

(3) an odd integer strictly greater than 1

(4) either 1 or 2

24. The trailing number of zeros (i.e., the number of zero at the end) in the number 201 ! is :

(1) 0 (2) 49

(3) 201 (4) 51

25. The last two digits in the number $123^{123}$ are :

(1) 67 (2) 69

(3) 27 (4) 89

26. Which of the following is *not* true for the Euler's phi function, $\varphi(n)$ ?

(1) $\varphi(mn) = \varphi(m)\,\varphi(n)\ \forall\ m, n \in \mathbf{N}$

(2) $\varphi(n)$ is even for all odd numbers except 1

(3) $\varphi(n)$ is even for all odd prime numbers

(4) $\varphi(p^n) = p^n - p^{n-1}$ for any prime number $p$ and any positive integer $n$

27. If $p$ is an odd prime number, then $(p + 2)\ [(p - 3)!] \equiv$

(1) $0 \pmod p$ (2) $1 \pmod p$

(3) $-1 \pmod p$ (4) $-3 \pmod p$

28. If $43^{31} \equiv n (\text{mod } 231)$, then $n =$

(1) 31 (2) 43

(3) 0 (4) 1

29. Let G be a group and H be a subgroup of G. Which of the following need not imply that H is a normal subgroup of G ?

(1) H is equal to the intersection of two normal subgroups of G

(2) Index of H in G is 2

(3) H is a normal subgroup of K, where K is a normal subgroup of G

(4) H is the Kernel of a group homomorphism from G to G

30. Let G be a group. Which of the following is *not* sufficient to say that G is an abelian group ?

(1) The map $f : G \to G$, defined as $f(x) = x^{-1}$ is a homomorphism

(2) The map $f : G \to G$, defined as $f(x) = x^2$ is a homomorphism

(3) Order of G is $p^2$, for some prime number $p$

(4) Order of G is $p^q$, for some prime numbers $p$ and $q$

31. Let G be an infinite cyclic group. The number of distinct proper subgroups of G that are isomorphic to G is :

(1) 0 (2) 1

(3) 2 (4) infinitely many

32. Let G be a group of order 6 and let $n$ be the number of elements of order 2 in G. A possible value of $n$ is :

(1) 0 (2) 3

(3) 2 (4) 4

33. The order of the subgroup generated by the element 10 (mod 12) in the group $\mathbf{Z}_{12}$ is :

(1) 3 (2) 4

(3) 6 (4) 12

34. The number of groups, up to isomorphism, of order 361 is :

(1) 2 (2) 19

(3) 1 (4) 38

35. Let G be a group of order 56 containing an even number of Sylow 7 – subgroups. The number of normal subgroups of order 7 in G is

(1) 0 (2) 8

(3) 1 (4) 7

36. Let $M_2(\mathbf{R})$ be the set of $2 \times 2$ matrices whose entries are real numbers and let $L = \left\{ \begin{pmatrix} a & 0 \\ c & d \end{pmatrix} / a, c, d \in \mathbf{R} \right\}$. Consider $M_2(\mathbf{R})$ and L as groups with respect to the matrix addition.

If $h : M_2(\mathbf{R}) \to L$ is defined as $h\left( \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right) = \begin{pmatrix} a+c & 0 \\ c+d & d+b \end{pmatrix}$, then the kernel of $h$ is :

(1) $\left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \right\}$ (2) $\left\{ \begin{pmatrix} x & -x \\ -x & x \end{pmatrix} / x \in \mathbf{Q} \right\}$

(3) $\left\{ \begin{pmatrix} -n & n \\ n & -n \end{pmatrix} / n \in \mathbf{N} \right\}$ (4) $\left\{ \begin{pmatrix} x\sqrt{2} & -x\sqrt{2} \\ -x\sqrt{2} & x\sqrt{2} \end{pmatrix} / x \in \mathbf{R} \right\}$

37. If $d$ is a square free positive integer, then the number of units in $\mathbf{Z}\left[\sqrt{-d}\right]$ is :

(1) 2 (2) 4

(3) either 2 or 4 (4) either 0 or 2

38. Let R be a non-zero commutative ring with unity. If M is a maximal ideal in R, then :

(1) R/M = (0)

(2) R/M is a field

(3) M + I = R for any ideal I in R

(4) M + I = M for any ideal I in R

39. The number of ring homomorphisms from $\mathbf{Z}_{2018}$ to $\mathbf{Z}_{2019}$ is :

(1) 0
(2) 1
(3) finite but greater than 1
(4) infinitely many

40. In the ring $\mathbf{Z}\left[\sqrt{-5}\right]$, the element $2+\sqrt{-5}$ is :

(1) irreducible but not prime
(2) prime but not irreducible
(3) both prime and irreducible
(4) neither irreducible nor prime

41. The polynomial $2x^2 + x + 2$ is irreducible over :

(1) $\mathbf{Z}_{19}$ but not $\mathbf{Z}_7$
(2) $\mathbf{Z}_7$ but not $\mathbf{Z}_{19}$
(3) both $\mathbf{Z}_7$ and $\mathbf{Z}_{19}$
(4) none of the fields $\mathbf{Z}_7$ and $\mathbf{Z}_{19}$

42. Consider the following field extensions :

(I) : $\mathbf{R}$ over $\mathbf{Q}$

(II) : $\mathbf{Q}\left(\sqrt{2}, \sqrt{3}, \ldots\ldots\right)$ over $\mathbf{Q}$, where $\mathbf{Q}\left(\sqrt{2}, \sqrt{3}, \ldots\ldots\right)$ is the smallest subfield of $\mathbf{R}$ containing $\mathbf{Q}$ and the square roots of all the positive prime numbers.

Which of these extensions is an infinite non-algebraic extension ?

(1) Only I
(2) Only II
(3) Neither I nor II
(4) Both I and II

43. Let $\alpha = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)$. The field extension $\mathbf{Q}(\alpha)$ over $\mathbf{Q}$ is :

(1) normal as well as separable
(2) normal but not separable
(3) separable but not normal
(4) neither normal nor separable

44. Let E be a field extension over F and let $a, b \in$ E which are algebraic over F. Let $m$ and $n$ be the degrees of the extensions, F($a$) and F($b$) over F respectively such that $m$ and $n$ are coprime. Then, the degree of the extension F($a$, $b$) over F is :

(1) $m + n$
(2) $mn$
(3) g.c.d. of $m$ and $n$
(4) $m^n$

45. Let $V = \mathbf{R}^2$ be a vector space with the following addition $\oplus$ and scalar multiplication $\odot$. $(x, y) \oplus (w, z) = (x + w + 1, y + z - 2)$ and $a \odot (x, y) = (ax + a - 1, ay - 2a + 2)$. The additive inverse of $(2, 3)$ is :

(1) $(-2, -3)$ (2) $(2, -3)$

(3) $(-4, 1)$ (4) $(-4, -1)$

46. Let $C(\mathbf{R})$ denote the vector space of all continuous real valued functions defined on $\mathbf{R}$, with respect to the usual addition and scalar multiplication. If $f(x) = 2x^2 + 3x + c$ for some $c \in \mathbf{R}$, then for which values of $c$, the vectors $f(x)$ and $f'(x)$ are linearly independent in $C(\mathbf{R})$ ?

(1) Any $c \in \mathbf{R}$ (2) Only for $c = 0$

(3) Only for $c > 0$ (4) Only for $c < 0$

47. Let V be the vector space of all $n \times n$ matrices over real numbers and W be the subspace of all symmetric matrices over real numbers. Then the dimension of W is :

(1) $n^2$ (2) $\frac{n(n-1)}{2}$

(3) $\frac{n(n+1)}{2}$ (4) $(n-1)^2$

48. Which of the following forms a basis for the null space of the matrix :

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 3 & 3 \end{bmatrix}$$

(1) $\left\{\begin{bmatrix} -3 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ 1 \\ 0 \end{bmatrix}\right\}$ (2) $\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}\right\}$

(3) $\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\}$ (4) $\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right\}$

49. Let $P_2$ be the vector space of all polynomials in $x$, with degree at most 2, with respect to the usual addition and scalar multiplication. The coordinate vector of the vector $5x^2 + 3x + 2$ with respect to the ordered basis, $\{x^2 + 1, x + 1, x^2 + x\}$ is :

(1) (2, 0, 3) (2) (3, 0, 2)

(3) (1, –1, 4) (4) (4, –1, 1)

50. Let $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$. Which of the following is an inner product on $\mathbf{R}^3$ ?

(1) $<u, v> = u_1v_1 + u_3v_3$

(2) $<u, v> = {u_1}^2{v_1}^2 + {v_2}^2{v_2}^2 + {u_3}^2{v_3}^2$

(3) $<u, v> = 2u_1v_1 + u_2v_2 + 4u_3v_3$

(4) $<u, v> = u_1v_1 - u_2v_2 + u_3v_3$

51. A basis for the orthogonal complement of the subspace of $\mathbf{R}^3$ spanned by the vectors $v_1 = (2, 0, -1)$ and $v_2 = (4, 0, -2)$ is

(1) $\{(1,0,0), (0,2,0)\}$ (2) $\left\{(0, 1, 0), \left(\frac{1}{2}, 0, 1\right)\right\}$

(3) $\{(2, 0, 0), (0, 3, 0), (0, 0, 5)\}$ (4) $\left\{\left(\frac{1}{2}, 0, 1\right), (4, 0, 8)\right\}$

52. Let W be the subspace of $\mathbf{R}^3$ spanned by the vectors (0,1,0), $\left(\frac{-4}{5}, 0, \frac{3}{5}\right)$. The orthogonal projection of (1,1,1) on W is

(1) $\left(\frac{4}{25}, 1, \frac{-3}{25}\right)$ (2) $\left(\frac{21}{25}, 0, \frac{28}{25}\right)$

(3) (4, 1, –3) (4) (21, 0, 28)

53. Which of the following linear transformations, T is one-one ?

(1) Any $T : \mathbf{R}^3 \to \mathbf{R}^2$ (2) $T(x, y) = (x + y, x + y)$

(3) $T(x, y) = (x + 2y, x - y)$ (4) $T(x, y) = (5x + 10y, 2x + 4y)$

54. Let T : $\mathbf{R}^2 \to \mathbf{R}^3$ be a linear transformation defined by $T(x, y) = (-x - y, 3x + 8y, 9x - 11y)$. Then rank and nullity of T, respectively are :

(1) 2 and 0 (2) 1 and 0

(3) 1 and 1 (4) 0 and 2

55. Given that –1 is an eigenvalue of $A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$. The geometric multiplicity of –1 is :

(1) 0 (2) 2

(3) 1 (4) 3

56. Given that –2 is an eigenvalue of $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$. Dimension of the eigenspace corresponding to $\lambda = -2$ is :

(1) 0 (2) 1

(3) 2 (4) 3

57. Let T : $\mathbf{R}^2 \to \mathbf{R}^3$ be a linear transformation given by $T((x_1, x_2)) = (x_2, -5x_1 + 13x_2, -7x_1 + 16x_2)$. The matrix of the transformation T with respect to the ordered bases.

B = {(3, 1), (5, 2)} for $\mathbf{R}^2$ and $B^1$ = {(1, 0, –1), (–1, 2, 2), (0, 1, 2,)} for $\mathbf{R}^3$ is :

(1) $\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$ (2) $\begin{bmatrix} 2 & 0 & 3 \\ 5 & 6 & 4 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$ (4) $\begin{bmatrix} 5 & 4 \\ 3 & 1 \\ 2 & 3 \end{bmatrix}$

58. Let B be the standard basis for $\mathbf{R}^n$ and let C = $\{c_1, c_2, \ldots\ldots c_n\}$ be another ordered basis. Writing $c_1, c_2, \ldots\ldots c_n$ as column vectors, the transition matrix from B to C is :

(1) $\begin{bmatrix} 1 & 0\ldots & 0 \\ 0 & 1\ldots & 0 \\ 0 & 0\ldots & 1 \end{bmatrix}$

(2) $[C_1, C_2, \ldots\ C_n]$

(3) $\begin{bmatrix} 2 & 0\ldots & 0 \\ 0 & 2\ldots & 0 \\ 0 & 0\ldots & 3 \end{bmatrix}$

(4) $[C_1, C_2, \ldots\ C_n]^{-1}$

59. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then 54 I + 145 A is

(1) $A^3$

(2) $A^5$

(3) $A^4$

(4) $A^6$

60. The matrix $A = \begin{pmatrix} 2 & 6 \\ 6 & y \end{pmatrix}$ is positive definite when :

(1) $y > 18$

(2) $y < 18$

(3) $y \geq 18$

(4) $y \leq 18$

61. Let X be a finite dimensional normed linear space. Then which of the following is *not* true ?

(1) Every linear operator T on X is bounded

(2) A subset $M \subseteq X$ is compact if and only if M is closed and bounded

(3) X is complete

(4) Every subspace Y of X need not be closed

62. Which of the following normed spaces has an inner product inducing the respective norm ?

(1) $(C([a, b]), \| \|_\infty)$ where $\| f \|_\infty = \sup_{x \in [a,b]} |f(x)|$

(2) $(l^3, \| \|_3)$ where $\| a_n \|_3 = \left(\sum_{n=1}^{\infty} |a_n|^3\right)^{1/3}$

(3) $(L^1[a, b], \| \|_1)$ where $\| f \|_1 = \int_a^b |f(x)| dx$

(4) $(l^2, \| \|_2)$ where $\| a_n \|_2 = \left(\sum_{n=1}^{\infty} |a_n|^2\right)^{1/2}$

63. Let X denote the linear space of all polynomials in one variable with coefficients in **R**. For $p \in X$ with $p(t) = a_0 + a_1 t + a_2 t^2 + \ldots\ldots + a_n t^n$, let :

$$\| p \| = \sup \{|p(t)| / 0 \le t \le 1\}$$

$$\| p \|_1 = |a_0| + |a_1| + \ldots\ldots + |a_n|$$

$$\| p \|_\infty = \max\{|a_0|, |a_1|, \ldots\ldots, |a_n|\}.$$

Then, which of the following is *true* for every $p \in X$ ?

(1) $\| p \|_1 \le \| p \|$, $\| p \|_\infty \le \| p \|_1$

(2) $\| p \|_1 \le \| p \|$, $\| p \|_1 \le \| p \|_\infty$

(3) $\| p \| \le \| p \|_1$, $\| p \|_1 \le \| p \|_\infty$

(4) $\| p \| \le \| p \|_1$, $\| p \|_\infty \le \| p \|_1$

64. The dual of $l^p$, $1 \le p < \infty$ is :

(1) $l^q$ with $\frac{1}{p} + \frac{1}{q} = 2$

(2) $l^q$ with $\frac{1}{p} + \frac{1}{q} = 1$

(3) $l^q$ with $\frac{1}{p} + \frac{1}{q} = 0$

(4) $l^p$

65. If $\varphi$ is a bounded linear functional on a Hilbert space H, then there exists a unique $u \in$ H such that for every $v \in$ H, $\varphi(v) = \langle v, u \rangle$ and $\| \varphi \| = \| u \|_H$. This is known as :

(1) Hahn-Banach theorem

(2) Closed Graph theorem

(3) Riesz representation theorem

(4) Banach Fixed point theorem

66. The space $C^1[a, b]$ (space of continuously differentiable functions on $[a, b]$) is a Banach space with the norm :

(1) $\| f \|_{c^1} = \| f \|_\infty$

(2) $\| f \|_{c^1} = \| f \|_\infty + \| f' \|_\infty$

(3) $\| f \|_{c^1} = \int_a^b | f | \, dx$

(4) $\| f \|_{c^1} = \left( \int_a^b | f(x) |^2 \, dx \right)^{1/2}$

67. Which of the following theorems is related to the extension of linear functionals on infinite dimensional vector spaces ?

(1) Uniform boundedness theorem

(2) Open mapping theorem

(3) Closed graph theorem

(4) Hahn-Banach theorem

68. Let U and V be two normed linear spaces. The space of all continuous linear operators from U into V is complete, if :

(1) U is complete

(2) V is complete

(3) Dim (V) = $\infty$

(4) Neither U nor V is complete

69. The analytic function $f(z) = u + iv$, such that $u - v = e^{2x}(\cos 2y - \sin 2y)$, is :

(1) $e^{2z} + e^{z} + c$ (2) $e^{z^2} + c$

(3) $e^{-2z} + c$ (4) $e^{2z} + c$

70. The radius of convergence of the Taylor series expansion of the complex function $f(z) = \dfrac{1}{(z+1)^2 (z+2)^2}$, about $z = 1$, is :

(1) 1 (2) 2 (3) 3 (4) 4

71. The value of the complex integral $\int_C \tan z \, dz$, where C is $|z| = 5$, is :

(1) $-4\pi i$ (2) $-8\pi i$ (3) $8\pi i$ (4) 0

72. Let $f(z) = a + bz + cz^2$ and $\oint_C \dfrac{f(z)}{z} dz = 4\pi i$, $\oint_C \dfrac{f(z)}{z^2} dz = 6\pi i$, $\oint_C \dfrac{f(z)}{z^3} dz = 8\pi i$, where C is $|z| = 1$. Then, the values of $a$, $b$ and $c$ respectively, are :

(1) 2, 3 and 4 (2) 3, 2 and 3

(3) 1, 1 and 1 (4) 1, 2 and 1

73. The value of the complex integral $\int_C z^3 e^{1/z^2} dz$, where C is $|z| = 1$, is :

(1) $2\pi i$ (2) $\pi i$ (3) $\pi i/2$ (4) 0

74. Under the bilinear transformation $w = u + iv = \dfrac{2z - 1}{z - 2}$, the region $|z| < 1$ is mapped as :

(1) $u^2 - v^2 < 1$ (2) $|w - 1| < 1$

(3) $|w + 2| < 1$ (4) $|w| < 1$

75. The image of the hyperbola $x^2 - y^2 = 1$, under the transformation $w = re^{i\theta} = 1/z$, is :

(1) $r^2 = \cos(2\theta)$ (2) $r^2 = \cos\theta$

(3) $r = \cos(2\theta)$ (4) $r = \cos(4\theta)$

76. The value of the real integral $\int_{-\infty}^{\infty} \dfrac{x + 2}{(x^2 + 1)(x^2 + 4)} dx$, is :

(1) $\dfrac{2\pi}{3}$ (2) $\dfrac{\pi}{3}$

(3) $\dfrac{\pi}{4}$ (4) $\dfrac{\pi}{6}$

77. The functions 1, cos $x$, sec $x$ are linearly independent on the interval :

(1) $(0, \infty)$ (2) $\left(0, \frac{\pi}{2}\right)$

(3) $\left[0, \frac{\pi}{2}\right]$ (4) $(0, \pi)$

78. The singular solution of the differential equation $p^2 + 2xp - y = 0$, where $p = \frac{dy}{dx}$, is :

(1) $x^2 + y^2 = 1$ (2) $x^2 - y^2 = 1$

(3) $x^2 = y$ (4) $x^2 + y = 0$

79. The orthogonal trajectories of the family of curves $r = k[\sec(2\theta) + \tan(2\theta)]$ are :

(1) $r = ce^{2\cos(2\theta)}$ (2) $r = ce^{-\cos(2\theta)}$

(3) $r = ce^{-[\sin(2\theta)]/4}$ (4) $r = ce^{-[\sin(2\theta)]/2}$

80. The differential equation $(x^2 + 2x - 3)^2 y'' + 3(x + 3)y' + (x - 1)y = 0$, is given. Then :

(1) $x = 1$ and $x = -3$ are regular singular points

(2) $x = 1$ is a regular singular point and $x = -3$ is an irregular singular point

(3) $x = 1$ is an irregular singular point and $x = -3$ is a regular singular point

(4) $x = 1$ and $x = -3$ are irregular singular points

81. The orthogonal trajectories of the family of curves $x^{4/3} + y^{4/3} = a^{4/3}$, are :

(1) $x^{2/3} - y^{2/3} = k^{2/3}$ (2) $x - y = k$

(3) $x^{4/3} - y^{4/3} = k^{4/3}$ (4) $x^{1/3} - y^{1/3} = k^{1/3}$

82. If the general solution of the differential equation $xy' = 4x^3(y - x)^2 + y$, is of the form $y = x + \frac{1}{z}$, then $z =$

(1) $\frac{k - x}{x}$ (2) $\frac{k - x^2}{x}$

(3) $\frac{k - x^3}{x}$ (4) $\frac{k - x^4}{x}$

83. The particular integral of the differential equation $y'' + 4y' + 3y = 130e^{2x}$ $\cos x$, is :

(1) $e^{2x}(7\cos x - 4\sin x)$ (2) $e^{2x}(4\cos x + 7\sin x)$

(3) $e^{2x}(\cos x + \sin x)$ (4) $e^{2x}(7\cos x + 4\sin x)$

84. The particular integral of the differential equation $y'' + y = \sec x$, is :

(1) $(\cos x)\ln(|(\sin x|) + x\cos x$

(2) $(\cos x)\ \ln(|\cos x) + x\sin x$

(3) $(\sin x)\ln(|\cos x|) + x\sin x$

(4) $(\sin x)\ln(|\sin x|) + x\sin x$

85. The general solution of the partial differential equation $2xyz + 2yzq = x^2 - y^2 + z^2$ is (where $\phi$ is an arbitrary $C^1$ function) :

(1) $x^2 + y^2 + z^2 = \phi(y/x)$ (2) $x^2 - y^2 - z^2 = x\phi(y/x)$

(3) $x^2 + y^2 + z^2 = x\phi(y/x)$ (4) $x^2 - y^2 + z^2 = y\phi(y/x)$

86. The general solution of the partial differential equation :

$$x(y^2 - z^2)p + y(z^2 - x^2)q - (x^2 - y^2)z = 0, \text{ is}$$

(where $\phi$ is an arbitrary $C^1$ function)

(1) $\phi(x^2 - y^2 + z^2, xyz) = 0$ (2) $\phi\left(\frac{x-y}{x+y}, xyz\right) = 0$

(3) $\phi\left(x^3 - y^3, \frac{x+y}{2z}\right) = 0$ (4) $\phi(x^2 + y^2 + z^2, xyz) = 0$

87. The general solution of the partial differential equation $2xyz = px^2y + qxy^2 + pq$, is given by (where $a$ and $b$ are arbitrary constants) :

(1) $z = ax^2 + by^2 + 2ab$ (2) $z = ax + by + 8ab$

(3) $z = ax^3 + by^3 + 8ab$ (4) $z = ax^2 + by^2 + 8ab$

88. Under the variables transformation $x = \frac{z + \bar{z}}{2}, y = \frac{z - \bar{z}}{2i}, u_{xx} + u_{yy} =$

(1) $\frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 u}{\partial \bar{z}^2}$ (2) $\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial \bar{z}^2}$

(3) $4\frac{\partial^2 u}{\partial z\,\partial \bar{z}}$ (4) $4\left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial u}{\partial \bar{z}}\right)$

89. The solution of the partial differential equation $yu_x - xu_y = 0$, is $u(x, y) =$ (where $c$ and $\lambda$ are arbitrary constants) :

(1) $ce^{\lambda(x^2 + y^2)/2}$ (2) $ce^{\lambda(x-y)/2}$

(3) $ce^{\lambda(x-y)}$ (4) $ce^{\lambda(x^2-y^2)/2}$

90. The differential equation $u_{xx} + 2xu_{xy} + (1 - y^2)u_{yy} + u_x + u_y + 5u = 0$, is classified as :

(1) a hyperbolic equation in $x^2 + y^2 > 1$

(2) a parabolic equation in $x > 0$

(3) a parabolic equation in $x^2 + y^2 > 1$

(4) an elliptic equation in $x^2 + y^2 < 4$

91. A particular integral of the partial differential equation $[D^2 - (D')^2]z = 12x^2 + 2y$, where $Dz = \frac{\partial z}{dx}$, $D'z = \frac{\partial z}{dy}$, is

(1) $x^4 + 2x^2y^2$ (2) $x^3 + 2x^2y$

(3) $x^4 + x^2y$ (4) $x^4 + x^2y^2$

92. The solutions of the differential equation $16\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial x^2}$, which decay with $y$, from the following is :

(1) $[Ax + B]e^{-k^2y}$, A, B $\in$ **R**, $k \in$ **N**

(2) $[A \cos(kx) + B \sin(kx)]e^{-4k^2y}$, A, B $\in$ **R**, $k \in$ **N**

(3) $[A \cos(4kx) + B \sin(4kx)]e^{-k^2y}$, A, B $\in$ **R**, $k \in$ **N**

(4) $[A \cos(4kx) + B \sin(4kx)]e^{-16k^2y}$, A, B $\in$ **R**, $k \in$ **N**

93. The complete integral of the partial differential equation $p^3 + q^3 - 27z = 0$, is (where $a$ and $b$ are arbitrary constants) :

(1) $(a^3 + 1)z^2 = 8(ax + y + b)^3$

(2) $(a^3 + 1)z = 8(ax + y + b)^3$

(3) $(a^3 + 1)z^2 = 8(ax + y + b)^2$

(4) $(a^3 + 1)z^2 = 8(ax + y + b)^4$

94. Which of the points P(1, –1, 3), Q(3, 3, 3) and R(1, 2, 3) lie on the same side of the plane $5x + 2y - 7z + 9 = 0$ ?

(1) Only P and Q

(2) Only Q and R

(3) Only P and R

(4) All P, Q and R

95. For each positive integer $n$, let $C_n$ be the collection of all those planes which make positive intercepts $a$, $b$ and $c$ on the X, Y and Z axes respectively such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = n$. Then :

(1) (0, 0, 0) lies on every plane in $C_n$, $\forall n \in \mathbf{N}$

(2) (1, 1, 1) lies on every plane in $C_n$, $\forall n \in \mathbf{N}$

(3) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ lies on every plane in $C_n$, $\forall n \in \mathbf{N}$

(4) $\left(\frac{1}{n}, \frac{1}{n}, \frac{1}{n}\right)$ lies on every plane in $C_n$, $\forall n \in \mathbf{N}$

96. If a tangent plane of the sphere $x^2 + y^2 + z^2 = 676$ makes intercepts $a$, $b$ and $c$ on the X, Y and Z axes respectively, then $\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right) =$

(1) 676

(2) $\frac{1}{676}$

(3) 26

(4) $\frac{1}{26}$

97. Let $S_1(x, y, z)$ and $S_2(x, y, z)$ be two polynomials of degree 2 such that $S_1 = 0$ and $S_2 = 0$ represent two spheres and the coefficients of $x^2$ in each of them is positive. Then :

(1) $S_1 + S_2 = 0$ and $S_1 - S_2 = 0$ represent orthogonal spheres

(2) $S_1 + S_2 = 0$ and $S_1 - S_2 = 0$ represent spheres but need not be orthogonal

(3) $S_1 + S_2 = 0$ represents a sphere but $S_1 - S_2 = 0$ need not represent a sphere

(4) $S_1 - S_2 = 0$ represents a sphere but $S_1 + S_2 = 0$ need not represent a sphere

98. The enveloping cone of the sphere $x^2 + y^2 + z^2 = 4$, with its vertex at (2, 2, 2) is given by :

(1) $x^2 + y^2 + z^2 = 2(xy + yz + zx) - 8$

(2) $x^2 + y^2 + z^2 = 2(xy + yz + zx) + 8$

(3) $x^2 + y^2 + z^2 = xy + yz + zx - 8$

(4) $x^2 + y^2 + z^2 = xy + yz + zx + 8$

99. Let $a$, $b$ and $c$ be positive real numbers. If the equation $ax^2 + by^2 + cz^2 + 2cyz + 2bxy + 2019 = 0$ represents a cone, then :

(1) $ab + ac + b^2 = 0$

(2) $ab + ac - b^2 = 0$

(3) $ab - ac - b^2 = 0$

(4) $ab - ac + b^2 = 0$

100. The equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and whose guiding curve is the ellipse $4x^2 + 9y^2 = 1$, $z = 3$ is :

(1) $4(3x - z + 3)^2 + (3y - 2z + 6)^2 = 1$

(2) $4(3x - z + 3)^2 + 9(3y - 2z + 6)^2 = 9$

(3) $4(3x - z + 3)^2 + (3y - 2z + 6)^2 = 9$

(4) $4(3x - z + 3)^2 + 9(3y - 2z + 6)^2 = 1$

## ROUGH WORK

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