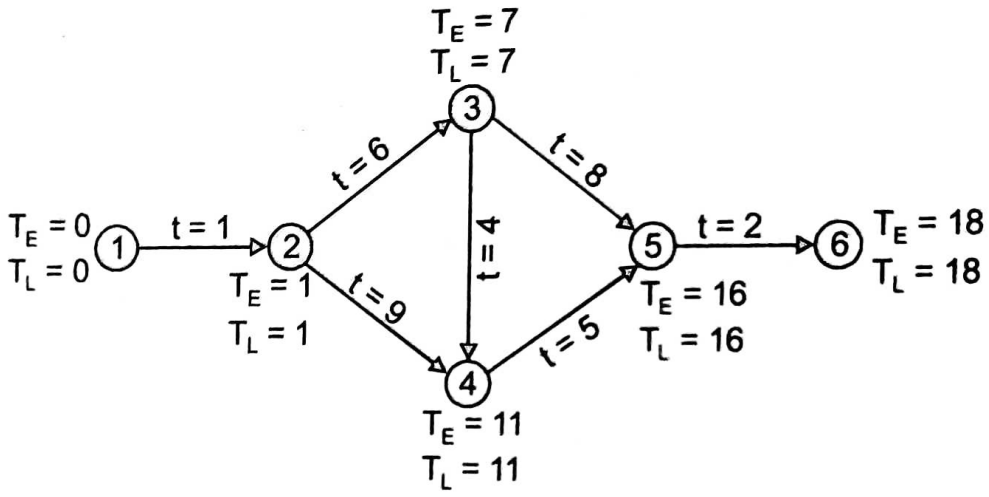
Q1. Match List-I (Item) with List-II (Characteristic) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Activity	1 Resourceless element
B. Event	2. Resource consuming element
C. Dummy	3. Spare time
D. Float	4. Instantaneous stage

Codes:

	A	B	C	D
(a)	1	3	4	2
(b)	2	1	4	3
(c)	2	4	1	3
(d)	3	4	1	2

Q2. The following diagram shows the details necessary for the CPM network analysis:



The critical path will be

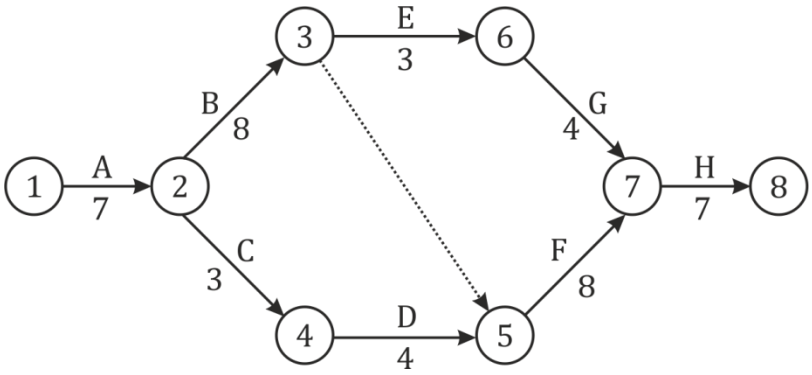
(a) 1 – 2 – 3 – 5 – 6

(b) 1 – 2 – 3 – 4 – 5 – 6

(b) 1 – 2 – 4 – 5 – 6

(b) 1 – 2 – 4 – 3 – 5 – 6

Q3.



The flow net of the activities of a project are shown in the network given above indicating the duration of the activities along their arrows. The critical path of the activities is along

(a) 1 – 2 – 4 – 5 – 7 – 8

(b) 1 – 2 – 3 – 6 – 7 – 8

(c) 1 – 2 – 3 – 5 – 7 – 8

(a) 1 – 2 – 4 – 5 – 3 – 6 – 7 – 8

Q4. Consider the following pairs:

1. Difference between total float and free float	:	Interfering float
2. Sum of independent float and tail slack	:	Free float
3. Sum of independent float, tail slack and interfering float	:	Total float

Which of these pairs are correctly matched?

(a) 1, 2 and 3

(b) 1 and 2

(c) 2 and 3

(d) 1 and 3

Q5. Which one of the following project management techniques is deterministic in nature?

(a) CPM

(b) PERT

(c) GERT

(d) LCES

Q6. What is the time by which the completion of an activity can be delayed without affecting the start of succeeding activities, called?

(a) Total float

(b) Interfering float

(c) Independent float

(d) Free float

Q7. Duration along the critical path defines which of the following?

1. Shortest duration needed

2. Shortest duration permissible

3. Longest duration needed

4. Longest duration permissible

Select the correct answer using the code given below:

(a) 1 and 2 only

(b) 1 and 4 only

(c) 2 and 3 only

(d) 3 and 4 only

Q8. Consider the following statements of network:

1. Only one time estimate is required for each activity.

2. Three time estimates for each activity.

3. Time and cost are both controlling factors.

4. It is built-up of event-oriented diagram.

Which of the above statements are correctly applicable to CPM network?

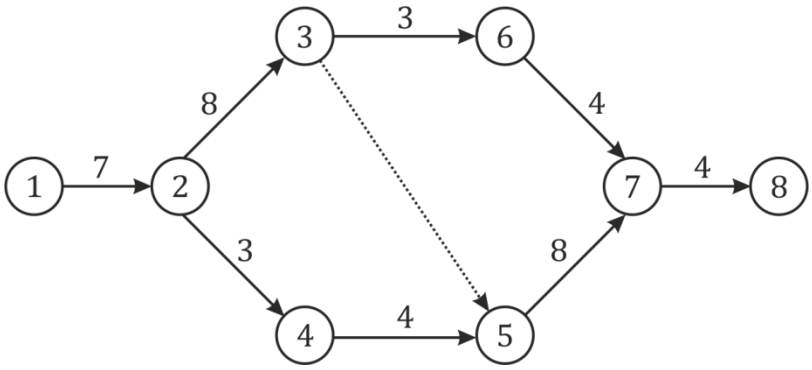
(a) 1 and 3

(b) 1 and 2

(c) 2 and 4

(d) 3 and 4 only

Q9. In the above network shown below, the duration of activities are written along their arrows. The critical path of the activities is along



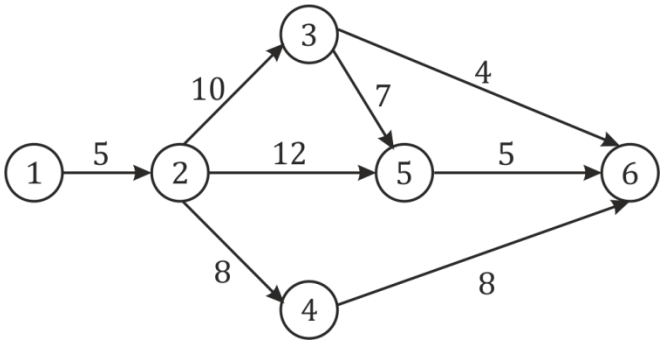
(a) 1 – 2 – 3 –5 – 7 – 8

(b) 1 – 2 – 3 –6 – 7 – 8

(c) 1 – 2 – 4 –5 – 7 – 8

(d) 1 – 2 – 3 – 4 – 5 – 7 – 8

Q10.



In the network shown above, the number on the arrow gives the duration of the activity. The earliest expected time for event 6 to be attained is

(a) 22

(b) 27

(c) 23

(d) 24

Q11. Critical path moves along the activities having total float as

(a) Positive

(b) Negative

(c) Zero

(d) Unity

Q12. Consider the following statements pertaining to CPM network analysis:

1. It is event-oriented method.

2. It is activity-oriented method.

3. Time and cost are controlling factors.

4. Time alone is controlling factor.

Which of these statements are correct?

(a) 1 and 2

(b) 2 and 3

(c) 3 and 4

(d) 1 and 4

Q13. In the critical path method of project planning, free float can be

(a) Greater than independent float

(b) Greater than total float

(c) Les than independent float

(d) Equal to total float

Q14. Which of the following statements are implicit in development the critical path network?

1. Only one time estimate is required for any activity

2. Time only is the controlling factor at this stage

3. Time and cost both are controlling factors at this stage

4. Critical events may have positive, negative, or zero float

(a) 1 and 2 only

(b) 1 and 3 only

(c) 1 and 4 only

(d) 2 and 4 only

Q15. Which of the following statements is/are correct for a non-critical activity?

1. It demands very special attention and action.

2. One can do with normal attention to this activity with some leeway for action.

Select the correct answer using the codes given below:

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Q16. The amount of time by which the start of the activity may be delayed without interfering with the start of any succeeding activity is called

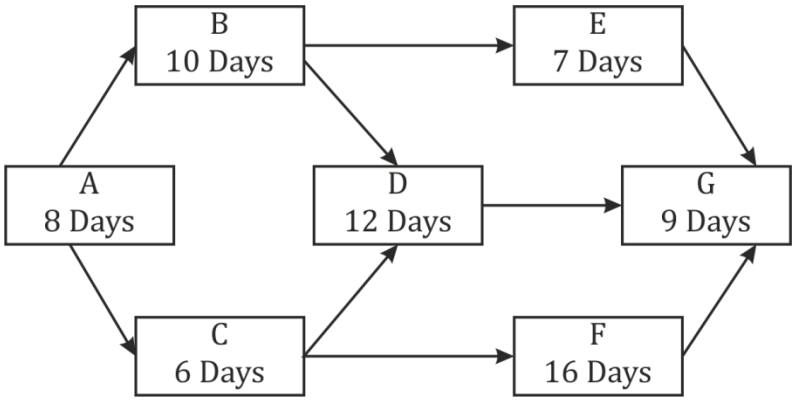
(a) Activity float

(b) Free float

(c) Total float

(d) Interfering float

Q17. A small project, consists of seven activities in the activity-on-node diagram as shown in the figure below. The duration of these activities, in days and the predecessor relationship are shown. What is the total project duration of the project?



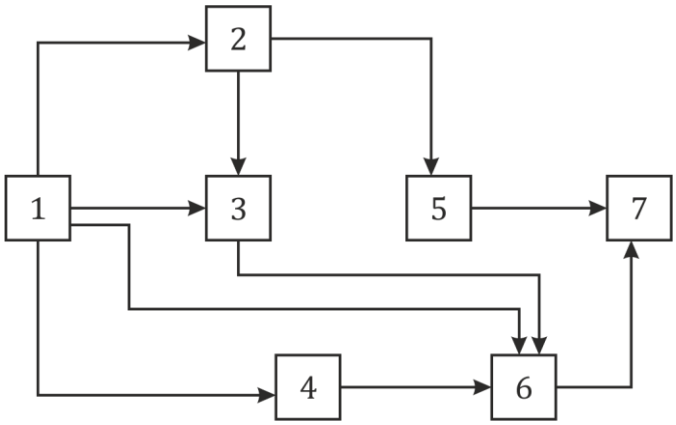
(a) 39 days

(b) 35 days

(c) 34 days

(d) 41 days

Q18. How many links are deletable in the A-O-N network shown below?



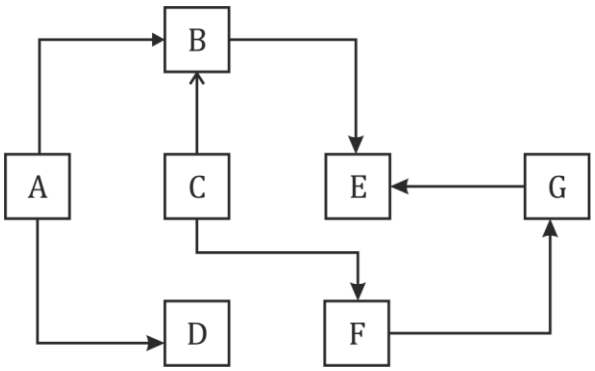
(a) None

(b) One

(c) Two

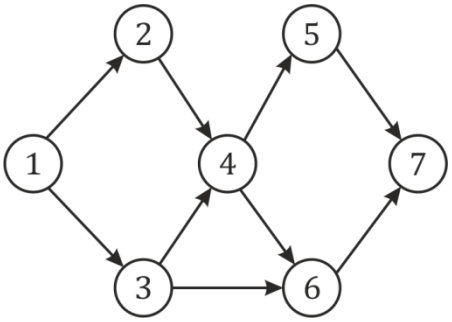
(d) Three

Q19. Consider the accompanying A-O-N diagram:

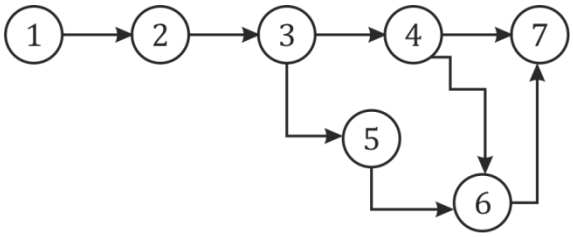


Which one of the following A-O-A diagrams correctly represents this A-O-N diagram?

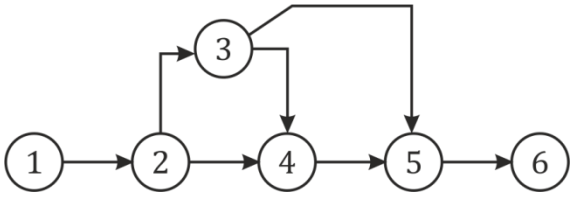
(a)



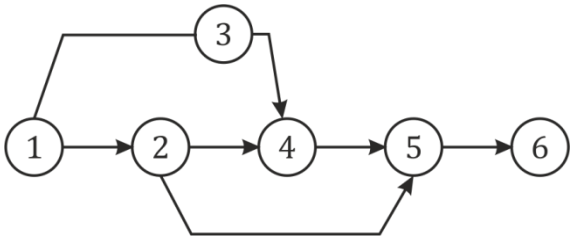
(b)



(c)



(d)



Q20. Which one of the following is relevant to activity on Node (AON)?

(a) Dummy activities may be many

(b) There will be no dummy activities

(c) It is used for quite complex project

(d) It is easier to associated with time flow of activities

Q21. Which system of network given below completely eliminates the use of dummy activities?

(a) A-O-A (Activity-on-Arrow)

(b) A-O-N (Activity-on-Node)

(c) PERT

(d) CPM

Q22. A serious limitation of inter dependences between various activities is generally observed in
(a) Bar charts
(b) Milestone charts
(c) Network analysis
(d) Job layouts

Q23. Match List – I (diagram Based Nomenclature) with List – II (information capability) and select the correct answer using the code given below the lists:

List – I
A. Work – breakdown structure B. Bar chart C. Linked bar chart D. Time computation on network
List – II
1. Target dates for interface events can be stipulated 2. Can be hierarchical 3. Can include information on cost distribution over time 4. Best suited for monitoring on network including that for costs

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	2	1	4	3
(c)	4	1	2	3
(d)	2	3	4	1

Q24. Consider the following statements in works breakdown structure:

1. It is a graphical representation of entire programmer.
2. The top-down approach to planning is adopted

3. The down-top approach to planning is adopted.
4. It is suitable for complex projects.

Which of the above statements is/are correct?
(a) 1 only
(b) 2 and 4
(c) 3 and 4
(d) 4 only

Q25. A bar chart is commonly used because
(a) It is simple to draw and easy to understand
(b) It indicates at a glance the overall progress of the project
(c) It shows critical and non-critical activities
(d) It incorporates uncertainties for delay in estimation of time required for completion of activates

Q26. A linked bar chart is an improvement over a conventional bar chart, because
1. resources for individual activities can be planned
2. floats will be available for utilization as needed
3. milestone events need not be specifically monitored

which of these is/are correct?
(a) 1, 2 and 3
(b) 3 only
(c) 2 only
(d) 1 only

Q27. The purpose of work – break –down structure in project planning is mainly to
1. Facilitate and improve the decision-making on procurement of resources
2. Relate activates under particular trade specialization to help in organizing for project staff
3. Co-ordinate regarding milestone events across trade specialization to improve the synergy between the trades

(a) 1 and 2 only
(b) 1 and 3 only
(c) 2 and 3 only
(d) 1, 2 and 3

Q28. Which of the following techniques belong to project time plan?
1. Critical path method
2. Precedence network analysis
3. Line of balance technique
4. Linear programmer chart

(a) 1, 2 and 3 only
(b) 1, 2 and 4 only
(c) 3 and 4 only
(d) 1, 2, 3 and 4

Q29. The graphical representations where in long duration jobs are broken down to key segmental elements, wherein events are shown in chronological order without attention to logical sequencing, and where in interdependencies between the events is not highlighted, is referred to as
(a) CPM
(b) Milestone chart
(c) GANTT chart
(d) PERT

Q30. Consider the following features/factors:
1. Projects are of the non-repetitive type
2. Time required need not be known
3. Time required is known precisely
4. Events have been established for planning
5. Emphasis is given to activities of project

PERT is preferred for planning because of
(a) 1, 2 and 4
(b) 3, 4 and 5
(c) 1, 3 and 4
(d) 1, 2 and 5

Q31. For a given activity, the optimistic time, pessimistic time and the most probable estimates are 5, 17 and 8 days respectively. The expected time is
(a) 8 days
(b) 9 days
(c) 10 days
(d) 15 days

Q32. The optimistic, most likely and pessimistic estimates of time for an activity are 4 days, 11 days and 12 days respectively. The expected completion time of this activity is
(a) 8 days
(b) 9 days
(c) 10 days
(d) 11 days

Q33. There are four consecutive activities in a simple linear network, each with mean duration of T and each with ‘k’ as the standard deviation of its duration. The overall project duration through these activities is likely to be in the range
(a) $4T \pm k$
(b) $4T \pm 2k$
(c) $4T \pm 4k$
(d) $4T \pm 6k$

Q34. Concreting at site can start on any one day, with all preparations having been done on the previous day. As per the past experience of owner, contractor and the architect, it is decided that the chance of any one being late is 0.4. the chance of starting on time on the appointed day is
(a) 0.064
(b) 0.216
(c) 0.288
(d) 0.432

Q35. In PERT analysis, the time estimates of activities and probability of their occurrence follow
(a) Normal distribution curve
(b) β – distribution curve
(c) Poisson's distribution curve
(d) Binomial distribution curve

Q36. The probability that the load on a scaffolding will exceed the design load of 3 tonnes is 0.15. at the time, the probability that the strength of the scaffolding will be more than 3 tonnes is 0.85. the probability that the scaffolding will fail is
(a) 0.2775
(b) 0.1275
(c) 0.0225
(d) 0.0020

Q37. Match List I (inputs into networks) with List – II (Basis) and select the correct answer:

List – I	List – II
A. Activity time	1. Availability of resources is not discussed
B. PERT durations	2. Senior managements involvement is assumed
C. WBS	3. Total cost of each activity is considered
D. Interfaces	4. Needed supervisory inputs are considered

Codes:

	A	B	C	D
(a)	3	1	4	2
(b)	3	1	2	4
(c)	4	2	1	3
(d)	4	2	3	1

Q38. The probabilistic time is
(a) $\frac{t_o+t_p+t_n}{3}$
(b) $\frac{t_o+t_p+4t_n}{6}$
(c) $\frac{t_o+4t_p+t_n}{6}$

(d) $\frac{t_o+2t_p+t_n}{4}$

Where,

t_o = optimistic time

t_p= pessimistic time

t_n = most likely time

Q39. If 'a' is the optimistic – time, 'b' is the pessimistic time, and 'm' is most likely time of activity then what is the expected time of activity?

(a) $\frac{a+m+b}{3}$

(b) $\frac{a+2m+b}{5}$

(c) $\frac{a+4m+b}{6}$

(d) $\frac{a+3m+b}{6}$

Q40. The optimistic, most and pessimistic time estimates of an activity are 5, 10 and 21 days respectively. What are the expected time and standard deviation?

(a) $12, 3$

(b) $11, 4$

(c) 11, 2.67

(d) 10, 16

Q41. Which of the following techniques is most suitable in case of research and development type of activity?

(a) Critical path method

(b) Project Evaluation and Review Technique

(c) Bar Chart

(d) Graphical Evaluation and Review Technique

Q42.Which one of the following relates to determination of critical path in PERT?

(a) Event oriented slack

(b) Activity oriented float

(c) Event oriented float

(d) Activity oriented slack

Q43. PERT calculations yield a project length of 60 weeks with a variance of 9 weeks. Probability factor corresponding to 95% probability is 1.674; then the number of weeks required to complete the project with a probability of 95% is

(a) 60.94

(b) 62.94

(c) 64.94

(d) 66.94

Q44. Consider the following statements:

1. PERT is activity-oriented and adopts deterministic approach.

2. CPM is event-oriented and adopts probabilistic approach.

3. PERT is event-oriented and adopts probabilistic approach.

Which of these statements is/are correct?

(a) 1 only

(b) 1 and 2

(c) 2 and 3

(d) 3 only

Q45. In PERT analysis, the time estimate of activities corresponds to

(a) Normal distribution

(b) Poisson's distribution

(c) β-distribution

(d) Binomial distribution

Q46. Slack time is associated with

(a) A real activity

(b) An event

(c) Both event and real activity

(d) Dummy activity

Q47. In PERT analysis, the time estimates of activities and probability of their occurrence follow:

(a) beta distribution

(b) gamma distribution

(c) normal distribution

(d) Poisson's distribution

Q48. PERT calculation yield a project length of 70 weeks with a variance of 9 weeks. The number of weeks required to complete the project with a probability of 95% (probability factor for 95% = 1.647) is

(a) 73.94 weeks

(b) 74.94 weeks

(c) 75.94 weeks

(d) 96.94 weeks

Q49. A father, notes that his teenage son uses the telephone. He takes no less than 5 minutes for a call and sometimes as much as an hour. 20 minutes calls are more frequent than calls of any other duration. Considered as a PERT activity a phone call's expected duration in minutes is

(a) $20\frac{1}{6}$

(b) $22\frac{1}{6}$

(c) $24\frac{1}{6}$

(d) $26\frac{1}{6}$

Q50. Slack time in PERT analysis

(a) Can never be greater than zero

(b) Is always zero for critical activities

(c) Can never be less than zero

(d) Is minimum for critical events

Solutions

S1. Ans. (c)

Sol. Activity: It is a resource consuming element.

Event: It is an instant stage or time at which some specified milestone is achieved.

Dummy: It is the activity which does not consume any resource.

Float: It denotes the time by which starting or finishing of an activity can be delayed without affecting the project completion time.

S2. Ans. (b)

Sol. Critical path in any network is the longest path time wise.

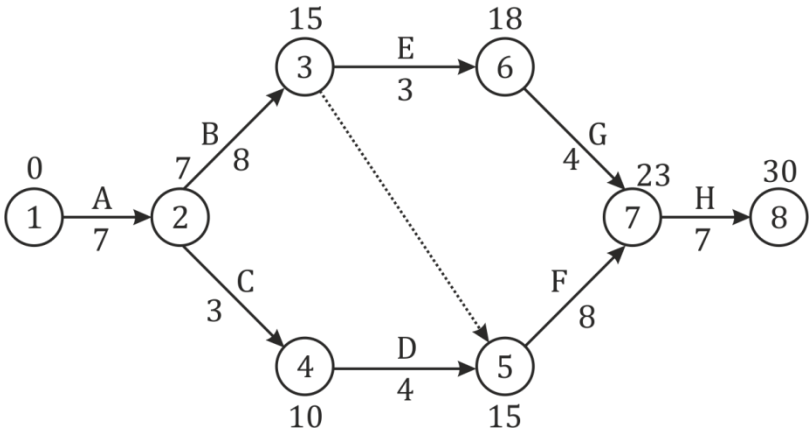
Hence for path

$1 - 2 - 3 - 4 - 5 - 6 = 18$ days,

Hence it is longest path time wise, so it becomes critical path.

S3. Ans. (c)

Sol. We know that critical path is the longest path time wise in any network.



For path

1 – 2 – 3 – 5 – 7 – 8, the time is 30 units. Hence it becomes critical path because it is longest path time wise.

S4. Ans. (a)

Sol. Interfering float – It is the Difference between total float and free float.

$F_{IT} = F_T - F_F$

Free float – It is the Sum of independent float and tail slack

$F_F = F_{ID} + S_i$

Total float – It is the Sum of independent float, tail slack and interfering float

$F_T = F_{IT} + F_F$

$F_T = F_{IT} + F_{ID} + S_i$

S5. Ans. (a)

Sol.

→ CPM is activity oriented and it follows deterministic approach.

→ PERT is event-oriented and adopts probabilistic approach.

S6. Ans. (d)

Sol. Free float is the time by which the completion of an activity can be delayed without affecting the start of succeeding activities.

S7. Ans. (c)

Sol. Critical path denotes the shortest permissible duration before which project cannot be completed and it also denotes the longest duration needed to complete the project.

S8. Ans. (a)

Sol. In CPM analysis:

→ It is an activity-oriented method.

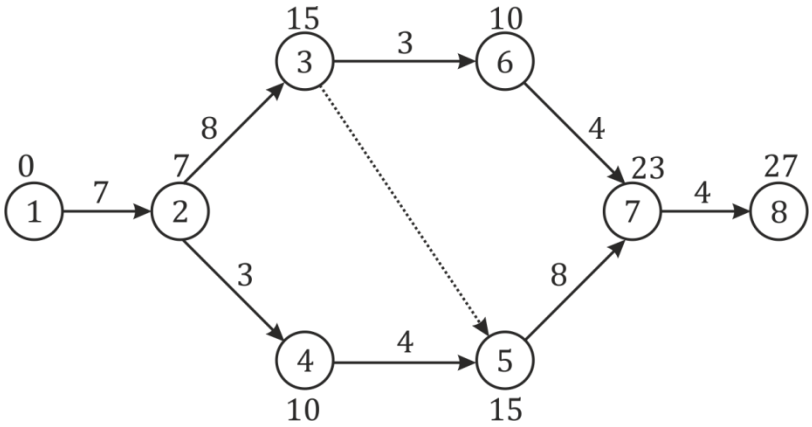
→ It follows deterministic approach.

→ In this, only one time estimate is required for each activity.

→ Time & Cost are controlling factors.

S9. Ans. (a)

Sol. We know that critical path is the longest path time wise in any network.

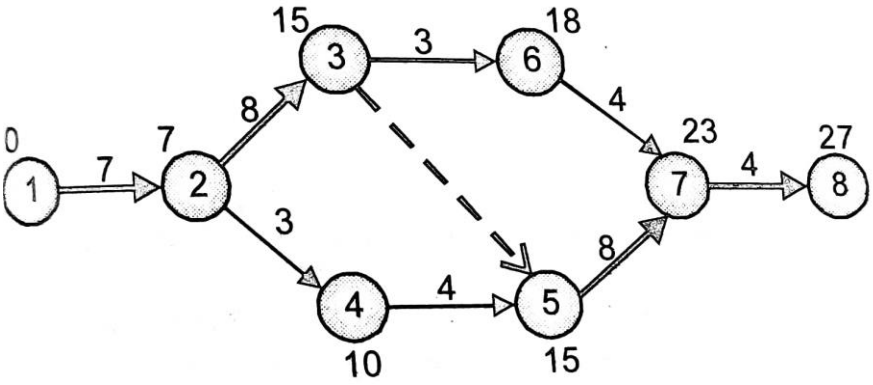


For path

1 – 2 – 3 – 5 – 7 – 8, the time is 27 units. Hence it becomes critical path because it is longest path time wise.

S10. Ans. (b)

Sol.



Hence, the earliest expected time for event 6 to be attained is 27 units.

S11. Ans. (c)

Sol. In CPM analysis, the critical path is determined by float approach. Critical path is the path along which total float of the activities is zero.

S12. Ans. (b)

Sol. In CPM analysis:

→ It is an activity-oriented method.

→ It follows deterministic approach.

→ In this, only one time estimate is required for each activity.

→ Time & Cost are controlling factors.

S13. Ans. (a)

Sol. Free float – It is the Sum of independent float and tail slack

$F_F = F_{ID} + S_i$

Total float – It is the Sum of independent float, tail slack and interfering float

$F_T = F_{IT} + F_F$

$F_T = F_{IT} + F_{ID} + S_i$

→ Free float will be greater than independent float.

→ Free float will be less than total float.

S14. Ans. (b)

Sol. In CPM analysis:

→ It is an activity-oriented method.

→ It follows deterministic approach.

→ In this, only one time estimate is required for each activity.

→ Time & Cost are controlling factors.

S15. Ans. (b)

Sol. Non critical activities can be done with normal attention.

S16. Ans. (b)

Sol. The amount of time by which the start of the activity may be delayed without interfering with the start of any succeeding activity is called Free float.

S17. Ans. (a)

Sol. The total duration of the project is 39 days along following paths.

(a) A – C –F – G = 39 days

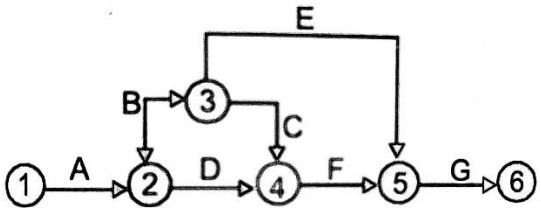
(b) A – B – D – G = 39 days

S18. Ans. (c)

Sol. There are two deletable links in the A-O-N network and there are 1 – 3 and 1– 6.

S19. Ans. (c)

Sol.



S20. Ans. (b)

Sol. Salient features about A-O-N diagram→

→ Events has no place.

→ This is not suitable for PERT.

→ Dummies have no place.

→ Interdependences is shown by arrow.

→ Nodes represents activities.

S21. Ans. (b)

Sol. Salient features about A-O-N diagram→

→ Events has no place.

→ This is not suitable for PERT.

→ Dummies have no place.

→ Interdependences is shown by arrow.

→ Nodes represents activities.

S22. Ans.(a)
Sol. Bar charts gives the idea about the physical project i.e., comparison of actual progress with the scheduled progress. In bar charts inter dependences between various activities is not shown.

S23. Ans.(d)
Sol. Work – breakdown structure - Work break down structure is hierarchical.

Bar chart – It can include information on cost distribution over time.

Linked bar chart – It is Best suited for monitoring on network including that for costs

Time computation on network - Target dates for interface events can be stipulated.

S24. Ans.(b)
Sol. Work break down structure is hierarchical. It follows top to down approach. It is the process of breaking the project into easily identifiable major systems, their sub- systems and discrete activities. It is suitable for complex projects.

S25. Ans.(a)

Sol. A bar chart is commonly used because it is simple to draw and easy to understand. Bar charts gives the idea about the physical project i.e., comparison of actual progress with the scheduled progress. In bar charts inter dependences between various activities is not shown.

S26. Ans.(c)
Sol. A linked bar chart is an improvement over a conventional bar chart, because in this, activities are linked with lines & arrows, specifying the sequence and order of preceding activities. Hence resource for whole and individual project can be planned. Information about float related with various activities can be determined.

S27. Ans.(d)
Sol. The purpose of work – break –down structure in project planning is mainly to

1. Facilitate and improve the decision-making on procurement of resources.
2. Relate activates under particular trade specialization to help in organizing for project staff.
3. Co-ordinate regarding milestone events across trade specialization to improve the synergy between the trades.

S28. Ans.(a)
Sol. Critical path method (CPM), Precedence network analysis and line of balance techniques are belonged to project time plan. While linear program chart is used to cost minimization, profit maximization and resources allocation.

S29. Ans.(b)
Sol. The graphical representations where in long duration jobs are broken down to key segmental elements, wherein events are shown in chronological order without attention to logical sequencing, and where in interdependencies between the events is not highlighted, is referred to as Milestone chart

S30. Ans.(a)
Sol. Salient features about PERT analysis is⟶

(i) Projects are of the non-repetitive type.
(ii) Time required need not be known.
(iii) Events have been established for planning.
(iv) PERT is usually suitable for research & development type projects.
(v) It follows probabilistic or non -deterministic approach.

S31. Ans.(b)
Sol. Expected time $(t_e) = \frac{t_o+4t_m+t_p}{6}$
Given,
Optimistic time (t_o) = 5
Most likely time (t_m) = 8
Pessimistic time (t_p) = 17
Now,

$(t_e) = \frac{5+(4\times8)+17}{6}$
= 9 days

S32. Ans.(c)
Sol. Expected time $(t_e) = \frac{t_o+4t_m+t_p}{6}$
Given,
Optimistic time (t_o) = 4
Most likely time (t_m) = 11
Pessimistic time (t_p) = 12
Now,
$(t_e) = \frac{4+(4\times11)+12}{6}$
= 10 days

S33. Ans.(d)
Sol. There are four consecutive activities in a simple linear network, each with mean duration of T and each with 'k' as the standard deviation of its duration. The overall project duration through these activities is likely to be in the range 4T ± 6k.

S34. Ans.(b)
Sol. The chance of starting on time on the appointed day is = (1 – 0.4) × (1 – 0.4) × (1 – 0.4)
= 0.216

S35. Ans.(b)
Sol. In PERT analysis, the time estimates of activities and probability of their occurrence follow β – distribution curve.

As there is not much information about the activities the 3-time estimates are taken which can be left skewed or right skewed.

S36. Ans.(c)
Sol. the probability that the scaffolding will fail is = 0.15 × (1 – 0.85)
= 0.0225

S37. Ans.(a)
Sol. Activity time - Total cost of each activity is considered.
PERT durations - Availability of resources is not discussed.
WBS - Needed supervisory inputs are considered.
Interfaces - Senior managements involvement is assumed.

S38. Ans.(b)
Sol. In PERT analysis the estimation of time estimates is done by probabilistic approach and expected time is calculated by weighted average method.

$$t_e = \frac{t_o + 4t_n + t_p}{6}$$

Where,
t_o = Optimistic time
t_p= Pessimistic time
t_n = Most likely time

S39. Ans.(c)
Sol. In PERT analysis the estimation of time estimates is done by probabilistic approach and expected time is calculated by weighted average method.

$$t_e = \frac{a + 4m + b}{6}$$

Where,
a = Optimistic time
b = Pessimistic time
m = Most likely time

S40. Ans.(c)
Sol. Given,

$$t_0 = 5$$
$$t_m = 10$$
$$t_p = 21$$
$$t_e = \frac{t_0 + 4t_m + t_p}{6}$$
$$= \frac{5 + (4 \times 10) + 21}{6} = \frac{66}{6} = 11\ days$$

Standard deviation (σ) $= \frac{t_p - t_o}{6}$

$$= \frac{21 - 5}{6}$$
$$= \frac{16}{6}$$

= 2.67 days

S41. Ans. (b)

Sol.

→ Project Evaluation and Review Technique (PERT) is most suitable in case of research and development type of activity.

→ Critical Path Method (CPM) is most suitable for repetitive type of projects.

S42. Ans. (a)

Sol.

→ In PERT analysis critical path is determined with the help of event-oriented slack.

→ Slack is the difference between the latest allowable occurrence time and earliest occurrence time of the head event of an activity.

S43. Ans. (c)

Sol. We know, $Z = \frac{T_S - T_E}{\sigma}$

Given, $T_E = 60$ weeks, $Z = 1.647$

$\sigma^2 = 9$

$\sigma = 3$

Now,

$1.647 = \frac{T_S - 60}{3}$

$T_S = 64.941$ weeks

S44. Ans. (d)

Sol.

→ CPM is activity oriented and it follows deterministic approach.

→ PERT is event-oriented and adopts probabilistic approach.

S45. Ans. (c)

Sol. In PERT analysis, the time estimates of activities and probability of their occurrence follow β – distribution curve.

As there is not much information about the activities the 3-time estimates are taken which can be left skewed or right skewed.

S46 Ans. (b)

Sol.

→ Slack time is associated with an event.

→ In PERT analysis critical path is determined with the help of event-oriented slack.

→ Slack is the difference between the latest allowable occurrence time and earliest occurrence time of the head event of an activity.

S47. Ans. (a)

Sol. In PERT analysis, the time estimates of activities and probability of their occurrence follow β – distribution curve.

As there is not much information about the activities the 3-time estimates are taken which can be left skewed or right skewed.

S48. Ans. (b)

Sol. We know, $Z = \frac{T_S - T_E}{\sigma}$

Given, $T_E = 70$ weeks

$Z = 1.647$

$\sigma^2 = 9$

$\sigma = 3$

Now, $1.647 = \frac{T_S - 70}{3}$

$T_S = 74.941$ weeks

S49. Ans. (c)

Sol. Given, $t_o = 5$

$t_m = 20$

$t_p = 60$

$t_E = ?$

$t_E = \frac{t_o + 4t_m + t_p}{6}$

$$= \frac{5 + (4 \times 20) + 60}{6}$$

$$t_E = 24\frac{1}{6}\text{min}.$$

S50. Ans. (d)

Sol. In PERT analysis

(a) $S > 0$ i.e., +ve

(b) $S < 0$ i.e., –ve

(c) $S = 0$ i.e., zero

→ for critical events slack will be minimum